Don't forget to measure Δs

or

Δs: There are things we know, and things we don't know…

- Programs at Bates, Mainz, and Jefferson Lab that plan to measure the strange electric and magnetic form factors depend on knowledge of the strange axial form factor -- but that form factor has never been directly measured!

- Connections between, and review of, available data linked to the strange axial form factor

- How to combine elastic νp and ep data to get G_A^s

- Combine BNL E734 νp data and the HAPPEX ep data to get 2 distinct solutions for the strange form factors at $Q^2 = 0.5$ GeV^2

- E734 and G^0 and FINeSSE

The nucleon axial form factor, polarized deep-inelastic scattering, and low-Q^2 neutral-current neutrino scattering: How are they all related?

They all involve matrix elements of the axial current: $\bar{q}\,\gamma_\mu\gamma_5\,q$

The nucleon axial form factor:

$$ {}_N\langle p'|\bar{q}\,\gamma_\mu\gamma_5\,q|p\rangle_N = \bar{u}(p')\gamma_\mu\gamma_5\,G_A^q(Q^2)u(p) $$

is the definition of the axial form factor $G_A^q(Q^2)$ for two nucleon states of momentum p and p'. $\left[Q^2 = -(p'-p)^2\right]$

The diagonal matrix elements of the axial current are the "axial charges"

$$ {}_N\langle p|\bar{q}\,\gamma_\mu\gamma_5\,q|p\rangle_N = 2Ms_\mu\Delta q $$

where M and s_μ are respectively the mass and spin vector of the nucleon. The quantities Δq are called the axial charges because they are the value of the axial form factor at $Q^2 = 0$.

Polarized Deep-Inelastic Scattering

The asymmetries measured in polarized deep - inelastic scattering,

$$A = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}$$

are related to the antisymmetric part of the virtual Compton amplitude, which contatins at its heart the axial current $\bar{q}\,\gamma_\mu \gamma_5\, q$. The structure function measured in these experiments, $g_1(x)$, is related to the axial charges:

$$\int_0^1 g_1(x)dx = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right] \qquad \text{(Bjorken 1966, 1970)}$$

Neutrino-Nucleon Elastic Scattering

The dominant contributors to low-Q^2 elastic neutrino scattering are the axial form factors. At $Q^2 = 0$, the cross section is given by:

$$\frac{d\sigma}{dQ^2}(\nu p \to \nu p) = \frac{G_F^2}{128\pi}\frac{M_p^2}{E_\nu^2}\left[\left(-\Delta u + \Delta d + \Delta s\right)^2 + \left(1 - 4\sin^2\theta_W\right)^2\right]$$

The strange axial form factor and Δs

During the early years of the "spin crisis" there was a great interest in Δs because it was thought to be the "solution" to the problem of the missing valence quark spin contribution to the proton spin.

By the mid 1990's it was clear that the complete "solution" involved also the gluon and orbital angular momentum sectors, and the focus of the spin community began to shift in those directions. The inclusive polarized DIS experiments produced useful (but assumption-laden) estimates of Δs.

The two major projects that could have measured Δs directly in neutrino scattering during this time period, the BNL E734 experiment and the LANL LSND experiment, failed to do so conclusively.

In the meantime, a program of parity-violating *eN* experiments arose, with the goal of measuring the strange *electromagnetic* form factors of the nucleon. This can't be done without knowing the strange axial form factor too, but since Δs had been "measured" in polarized DIS they wisely proceeded with their plans.

Seven experiments discussed in this talk (all fixed target)

1) SMC (CERN): measured inclusive deep inelastic scattering of polarized muons from polarized nuclear targets ($Q^2 \sim 10$ GeV^2)

2) HERMES (HERA): measured inclusive and semi-inclusive deep inelastic scattering of polarized positrons from polarized nuclear targets ($Q^2 \sim 2.5$ GeV^2)

3) SAMPLE (MIT/Bates): measured parity violating asymmetries in backward-angle electron-proton and electron-deuteron elastic scattering ($Q^2 = 0.091$ GeV^2)

4) HAPPEX (JLab): measured PV asymmetries in forward-angle electron-proton elastic scattering ($Q^2 = 0.477$ GeV^2)

5) G^0 (JLab): is measuring PV asymmetries in forward and backward electron-proton and electron-deuteron elastic scattering for $0.1 < Q^2 < 1.0$ GeV^2

6) E734 (BNL): measured neutrino-proton and antineutrino-proton elastic scattering cross section for $0.45 < Q^2 < 1.05$ GeV^2

7) FINeSSE (?): seeks to measure ratio of NC and CC neutrino elastic scattering for $0.2 < Q^2 < 1.0$ GeV^2

Note on quarks and anti-quarks

Elastic and inclusive lepton-nucleon measurements cannot distinguish between quark and anti-quark distributions. So, the axial charges measured in most of these experiments are actually sums of quark and anti-quark contributions.

Of the experiment discussed in this talk, only the HERMES semi-inclusive DIS measurements, which tag the quark flavor by detecting a leading hadron in coincidence with the scattered lepton, can separately measure quark and anti-quark polarized parton distribution functions.

Data on Δs from Inclusive Polarized Deep Inelastic Scattering

In the quark-parton model, inclusive scattering of leptons from nucleon targets measures the nucleon structure function F_1:

$$F_1(x) = \frac{1}{2}\sum_q e_q^2 q(x) \qquad q(x) = \text{p.d.f.}$$

Inclusive scattering of **polarized** leptons from **polarized** nucleon targets measures the **spin-dependent** nucleon structure function g_1:

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Delta q(x) \qquad \Delta q(x) = \text{ polarized p.d.f.}$$

The first moment of the polarized p.d.f. is the contribution of that flavor to the nucleon spin:

$$\Delta q \equiv \int_0^1 \Delta q(x)\,dx$$

The Δq are also called the “axial charges” because they are related to the matrix elements of the axial current:

$$\Delta q \propto \bar{q}\gamma_\mu \gamma_5 q$$

QCD: Q^2-dependence and radiative corrections

In leading order QCD, these functions take on a scale dependence:

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \Delta q(x,Q^2)$$

In NLO QCD, there are significant radiative corrections and the relation between g_1 and the Δq is more complex.

The discussion here will be limited to the leading-order QCD analysis for two reasons:

- Including the NLO terms does not change the answer for Δs very much.
- The problems I will point out exist at all orders, because they are problems coming from the data itself.

The classic LO QCD analysis of inclusive polarized DIS data

[SMC: Adeva *et al.*, Phys. Lett. B 412 (1997) 414]

1) Take measured values of $g_1(x, Q^2)$, covering ranges of x and Q^2 of $0.003 < x < 0.70$ and $1.3 < Q^2 < 58.0$, and use QCD to evolve all data to a common value of $Q^2 = 10$ GeV^2. The evolution procedure leads to a fit function for g_1 in the measured x-region.

2) Extrapolate the measured values to $x = 1$ assuming a constant value of the experimental asymmetry. (This step contributes little to the **area** of g_1 and also very little to the uncertainty and I don't discuss it again.)

3) Extrapolate the measured values to $x = 0$. It is unclear how to do this, so <u>two</u> approaches are used*. One is to assume g_1 = constant for $x < 0.003$ --- this is called the "Regge assumption" for not very good reasons. The other approach is to simply use the QCD fit from step 1, extended to $x = 0$.

*Dogs walk around in a circle <u>twice</u> before laying down….

What the data and fits look like:

The QCD fit in the region $x < 0.003$ contributes significantly to the area of g_1 because g_1 is **large** there. (It is usual to plot xg_1 to make a sensible-looking picture.)

For example, at $x = 10^{-4}$, $xg_1 \sim -.002$, so $g_1 \sim -20$. You can do the math.

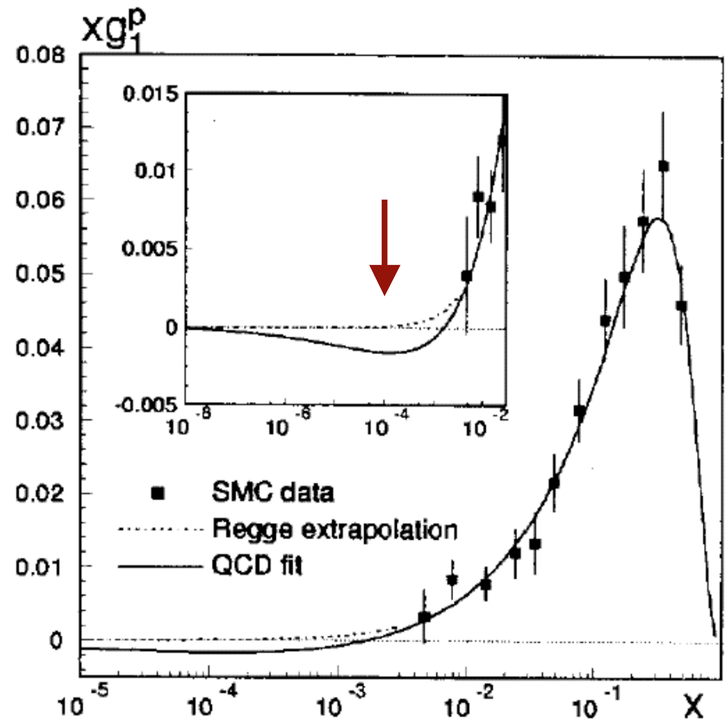


Fig. 5. xg_1^p as a function of x; SMC data points (squares) with the total error are shown together with the result of the QCD fit (continuous line), both at $Q^2 = 10\ \mathrm{GeV}^2$. For $x < 0.003$ the extrapolation assuming Regge behaviour is indicated by the dot-dashed line. The inset is a close-up extending to lower x.

[SMC: Adeva *et al.*, Phys. Lett. B 412 (1997) 414]

Completing the analysis, using both fits:

4) Integrate g_1 over $0 < x < 1$: $\Gamma_1 = \int_0^1 g_1(x)dx = \begin{cases} 0.142 \pm 0.017 & \text{"Regge"} \\ 0.130 \pm 0.017 & \text{QCD fit} \end{cases}$

This integral relates the three axial charges:

$$\Gamma_1 = \int_0^1 g_1(x)dx = \frac{1}{2}\sum_q e_q^2 \int_0^1 \Delta q(x)dx = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$

5) Assuming $SU(3)_f$ is a valid symmetry of the baryon octet, and using hyperon beta decay data, we can determine two other relations between the three axial charges:

$$\Delta u - \Delta d = \frac{g_A}{g_V} = F + D \qquad \text{and} \qquad \Delta u + \Delta d - 2\Delta s = 3F - D$$

where $\frac{g_A}{g_V} = 1.2601 \pm 0.0025$ and $\frac{F}{D} = 0.575 \pm 0.016$ [in 1997]

6) Solve for the axial charges!

	"Regge"	QCD fit
Δu	0.84 ± 0.06	0.80 ± 0.06
Δd	-0.42 ± 0.06	-0.46 ± 0.06
Δs	-0.08 ± 0.06	-0.12 ± 0.06

Data on Δs from Semi-Inclusive Polarized Deep Inelastic Scattering

In semi-inclusive DIS, a leading hadron is observed in coincidence with the scattered lepton. This allows a statistical identification of the struck quark, and hence a measurement of the x-dependence of the individual $\Delta q(x)$ distribution functions. (Inclusive scattering only measures the total structure function $g_1(x)$.)

The HERMES experiment on the HERA ring at DESY was especially designed to make this measurement.

HERMES measured double-spin asymmetries in the production of charged hadrons in polarized deep-inelastic scattering of positrons from polarized targets: Specifically, the asymmetry in the production of charged pions on targets of hydrogen and deuterium, and of charged kaons in scattering from deuterium.

HERMES measurement of $\Delta q(x)$

There is no assumption of $SU(3)_f$ symmetry in their analysis. They extract the following quark polarization distributions, over the range $0.023 < x < 0.30$: $\dfrac{\Delta u}{u}(x) \quad \dfrac{\Delta d}{d}(x) \quad \dfrac{\Delta \bar{u}}{\bar{u}}(x) \quad \dfrac{\Delta \bar{d}}{\bar{d}}(x) \quad \dfrac{\Delta s}{s}(x)$

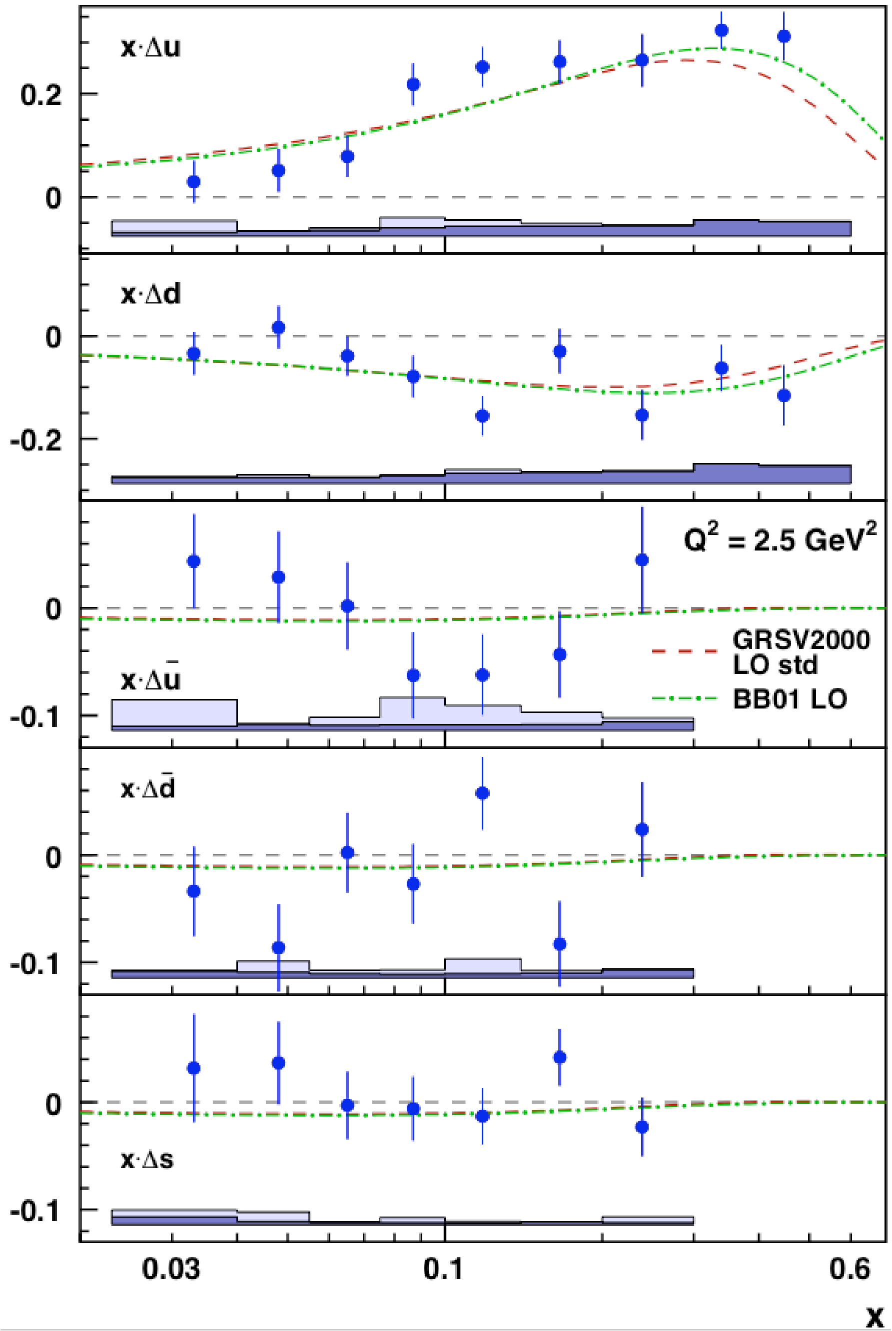
where $\dfrac{\Delta s}{s}(x)$ is defined to be the sum of $\dfrac{\Delta s}{s}(x)$ and $\dfrac{\Delta \bar{s}}{\bar{s}}(x)$.

They integrate the strange distribution to obtain:

$$"\Delta s" = \int_{x=0.023}^{0.30} \Delta s(x)\, dx = +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$$

$\left[\text{This would only be the true } \Delta s \text{ if the integral was over } 0 \le x \le 1.\right]$

HERMES Results: quark helicity distributions



PRL 92 (2004) 012005

u and *d* quarks

polarized

$$Q^2 = 2.5 \text{ GeV}^2$$

$$0.023 < x < 0.30$$

sea quarks

unpolarized

$$\text{"}\Delta s\text{"} = \int_{x=0.023}^{0.30} \Delta s(x)\, dx$$

$$= +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$$

So, where did the negative Δs go?

The HERMES article [PRL 92 (2004) 012005] contains this very interesting remark:

"The strange sea distribution was previously found to be negatively polarized in the analysis of only inclusive data assuming SU(3) symmetry applied to hyperon beta decay data. However, the first moments from such analyses evaluated over the measured x range $(\Delta s + \Delta\bar{s})/2 \equiv \int_{0.023}^{0.3} \Delta s(x)\, dx$ are typically not in disagreement with the partial moment of the density extracted here: $\Delta s = +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$."

Translation: If both analyses are correct, then all of the negative contribution to Δs from the $SU(3)_f$ analysis of inclusive DIS data came from lower values of x, that is from $x < 0.023$.

Low-x strange quark polarization

The HERMES result suggests that the strange quark helicity distribution $\Delta s(x) \sim 0$ for $x > 0.023$.

At the same time, we have our result from the SU(3)$_f$ analysis of hyperon beta decay and inclusive polarized DIS data, including an extrapolation to $x = 0$:

$$\Delta s = \int_0^1 \Delta s(x)\, dx \approx -0.10 \pm 0.06$$

If these two results are both true, then the average value of $\Delta s(x)$ in the range $x < 0.023$ must be ~ –5. That's not impossible, as $s(x)$ is ~20-300 in the range x~10^{-2} to 10^{-3} (CTEQ6). But we would need a mechanism that would "turn on" the strange quark polarization suddenly at these low x values.

Other explanations

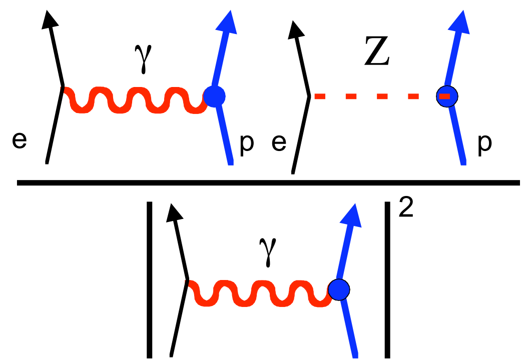
• SU(3)$_f$ symmetry is not valid. That would have been an unhappy conclusion a few years ago, when experimental data was sparse and we needed SU(3)$_f$ to extend the reach of the analysis. But with so much data available now, and more coming soon from RHIC, COMPASS, etc., perhaps we don't need to lean on SU(3)$_f$ as a crutch any more.

• The extrapolations to $x = 0$ are not valid. There is no way to prove or disprove this, except via additional experimental measurements.

A direct measurement of Δs would certainly serve to clarify some of these issues!

Parity Violating Electron Scattering

polarized electrons
unpolarized target



$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

for a nucleon:

$$= \left[\frac{-G_F Q^2}{4\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{\gamma} G_E^{Z} + \tau G_M^{\gamma} G_M^{Z} - (1 - 4\sin^2\theta_W)\varepsilon' G_M^{\gamma} G_A^{e}}{\varepsilon (G_E^{\gamma})^2 + \tau (G_M^{\gamma})^2}$$

$$\tau = \frac{Q^2}{4M^2}$$

$$\varepsilon = \left[1 + 2(1+\tau)\tan^2(\theta/2)\right]^{-1}$$

$$\varepsilon' = \sqrt{(1-\varepsilon^2)\tau(1+\tau)}$$

forward angles
HAPPEX, Mainz, G^0: sensitive to
G_E^s and G_M^s

backward angles
SAMPLE, G^0: sensitive to
G_M^s and G_A^e

effective axial-vector
e-N form factor

The effective axial form factor seen in parity-violating $\vec{e}N$ scattering

$$G_A^e = -G_A^{CC} + G_A^s + G_A^{\mathrm{ewm}}$$

$$G_A^{CC} = \frac{g_A}{\left(1 + Q^2/M_A^2\right)^2}$$

$$g_A = 1.267$$
$$M_A = 1.026\ \mathrm{GeV}$$

Charged-current axial form factor; dipole Q^2 behavior

$$G_A^s(Q^2=0) = \Delta s$$
$$= -0.10 \pm 0.06$$

[estimate from DIS data]

Strange quark axial form factor; unknown Q^2 behavior

G_A^{ewm} Contains **electroweak mixing** contributions ---- must be calculated…

Isoscaler and isovector components of $G_A{}^e$

The terms in G_A^{ewm}, being corrections, are either additive terms or are proportional to G_A^{CC} and G_A^s. It is customary to rewrite $G_A^e = -G_A^{CC} + G_A^s + G_A^{\text{ewm}}$ in terms of isoscaler and isovector components:

$$G_A^e = G_A^e(T=0) + G_A^e(T=1)$$

with $\quad G_A^e(T=0) = G_A^s + R_A^{T=0}$

and $\quad G_A^e(T=1) = -G_A^{CC}\left(1 + R_A^{T=1}\right)$

The correction factors $R_A^{T=0}$ and $R_A^{T=1}$ have been calculated at $Q^2 = 0$ in the context of heavy - baryon chiral perturbation theory by Zhu et al., Phys. Rev. D62 (2000) 033008.

$$R_A^{T=0} = 0.03 \pm 0.05 \qquad R_A^{T=1} = -0.23 \pm 0.24$$

Known form factors ($Q^2 < 1$ GeV^2)

$$G_E^p = \frac{1}{\left(1 + Q^2 / M_V^2\right)^2} \qquad M_V = 0.843 \text{ GeV} \qquad [\pm 2\%]$$

$$G_E^n = -\frac{\mu_n \tau}{1 + 5.6\tau} G_E^p \qquad \mu_n = -1.913 \qquad [\pm 20\%]$$

$$G_M^p = \frac{\mu_p}{\left(1 + Q^2 / M_V^2\right)^2} \qquad \mu_p = +2.793 \qquad [\pm 2\%]$$

$$G_M^n = \frac{\mu_n}{\left(1 + Q^2 / M_V^2\right)^2} \qquad [\pm 3\%]$$

$$G_A^{CC} = \frac{g_A}{\left(1 + Q^2 / M_A^2\right)^2} \qquad g_A = 1.267 \pm 0.035, \quad M_A = 1.026 \pm 0.021 \text{ GeV}$$

The SAMPLE Result

SAMPLE measured PV asymmetry in backward angle elastic $\vec{e}N$ scattering at $Q^2 = 0.091\ \text{GeV}^2$, using both hydrogen and deuterium targets:

$$A_{\text{H}} = -5.61 \pm 0.67 \pm 0.88\ \text{ppm} \quad \text{(Phys. Lett. B583 (2004) 79)}$$

$$A_{\text{D}} = -7.77 \pm 0.73 \pm 0.62\ \text{ppm} \quad \text{(Phys. Rev. Lett. 92 (2004) 102003)}$$

Assuming: $G_A^s \cong \Delta s = -0.1 \pm 0.1$ (E. Beise, priv. comm.),

$G_E^s = 0$ (this must be nearly zero near $Q^2 = 0$), and

the correction factor $R_A^{T=0} = 0.03 \pm 0.05$ (Zhu et al.),

then these two assymmetries are related to G_M^s and $G_A^e(T=1)$ as follows:

$$A_{\text{H}} = -5.56 + 3.37 G_M^s + 1.54 G_A^e(T=1)\ \text{ppm}$$

$$A_{\text{D}} = -7.06 + 0.77 G_M^s + 1.66 G_A^e(T=1)\ \text{ppm}$$

SAMPLE reports a measurement of G_M^s and $G_A^e(T=1)$:

$$G_M^s = 0.17 \pm 0.35 \pm 0.39 \qquad G_A^e(T=1) = -0.41 \pm 0.55 \pm 0.49$$

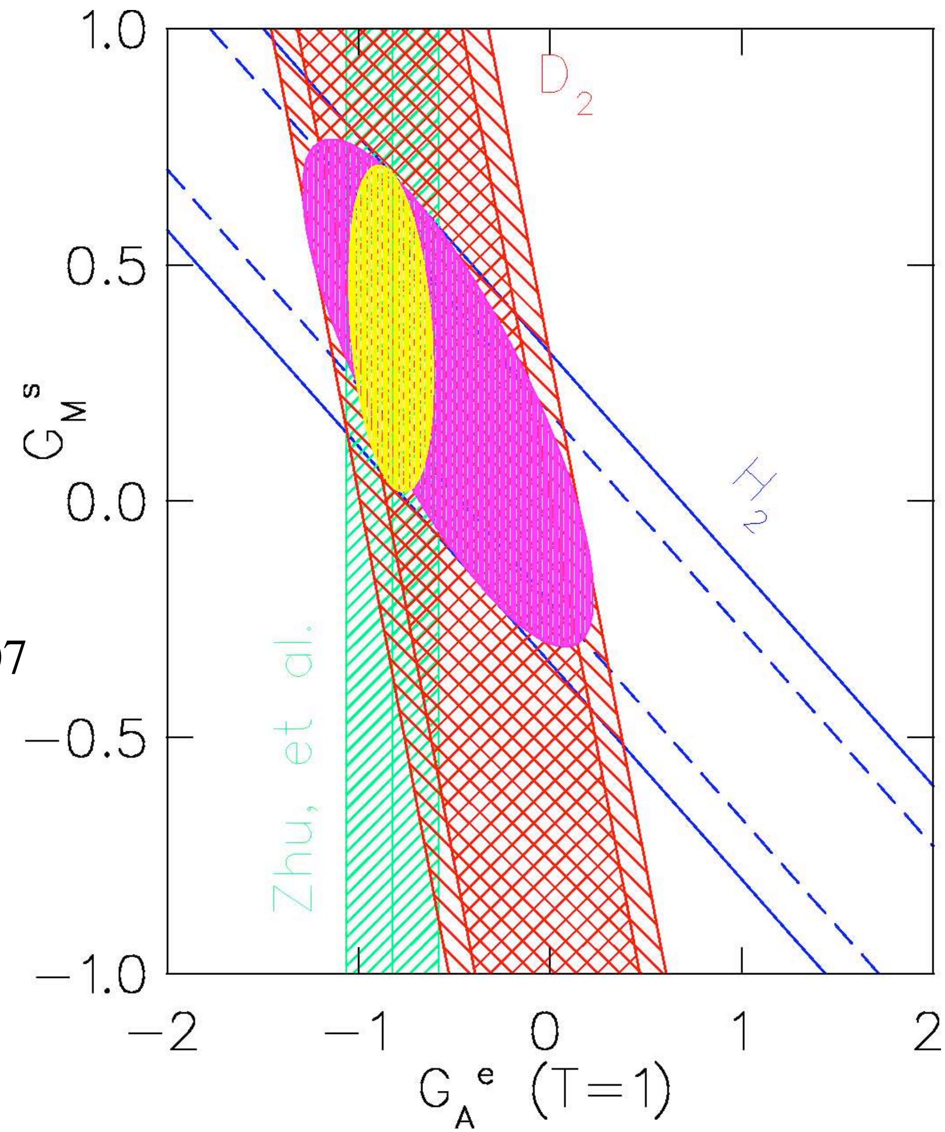
The reason for separating out the $G_A^e(T=1)$ term was to test the more difficult aspect of the Zhu et al. calculation, namely the isovector correction factor: $R_A^{T=1} = -0.23 \pm 0.24$. Using this value, Zhu et al. are able to predict $G_A^e(T=1) = -0.83 \pm 0.26$, in agreement with the experiment.

Making use of the theory…

If one believes the calculation of Zhu et al., one may use instead their value of $G_A^e(T=1)$ with the experimental value of A_H and extract a value for G_M^s :

$$G_M^s\left(Q^2 = 0.1\ \mathrm{GeV}^2\right) = 0.37 \pm 0.20 \pm 0.26 \pm 0.07$$

[SAMPLE : Phys. Lett. B583 (2004) 79]



But if you believe the calculation of the correction factors, why not discard the assumption of $G_A^s = -0.1 \pm 0.1$?

Instead, why not use the hydrogen and deuterium asymmetries to extract G_A^s along with G_M^s ?

But it doesn't work that way….

If you take $G_A^e(T=1)$ to be known, and try to let G_A^s and G_M^s be the two unknowns, you discover that they are related by the same slope in **both** the hydrogen and deuterium asymmetries.

$$\tau = \frac{Q^2}{4M^2}$$

$$\frac{\partial G_M^s}{\partial G_A^s} = -\left(1 - 4\sin^2\theta_W\right)\frac{\varepsilon'}{\tau}$$

$$\varepsilon = \left[1 + 2(1+\tau)\tan^2(\theta_e/2)\right]^{-1}$$

$$\varepsilon' = \sqrt{(1-\varepsilon^2)\tau(1+\tau)}$$

$$\approx -\frac{1}{2} \quad \text{for SAMPLE at } Q^2 = 0.091\ \text{GeV}^2$$

Back-angle PV *eN* asymmetry measurements can only establish a relationship between the strange magnetic and axial form factors, they cannot measure them both.

The present uncertainty in the strange axial form factor (*i.e.* Δs) is a significant contributor to the uncertainty in the strange magnetic form factor extracted from the SAMPLE data.

The HAPPEX Result

HAPPEX measured forward PV $\vec{e}N$ asymmetry at $Q^2 = 0.477\ \mathrm{GeV}^2$.

Their result is a linear combination of G_E^s and G_M^s :

$$G_E^s + 0.392 G_M^s = 0.025 \pm 0.020 \pm 0.014$$

This contains a contribution from G_A^e as well, so there may be some additional uncertainty in the result.

But their sensitivity to G_A^e is about 4% of that to G_E^s and G_M^s, because the measurement is at a very forward electron scattering angle. (No axial contribution at $\theta_e = 0$.) So even a large uncertainty in G_A^e at this Q^2 would not pose a significant problem in the interpretation of this result.

Forward-angle PV *eN* asymmetry measurements establish a relationship between the strange electric and magnetic form factors.

The BNL E734 Experiment

- performed in mid-1980’s
- measured neutrino- and antineutrino-proton elastic scattering
- used wide band neutrino and anti-neutrino beams of $<E_\nu>$=1.25 GeV
- covered the range $0.45 < Q^2 < 1.05$ GeV^2
- large liquid-scintillator target-detector system
- still the **<u>only</u>** elastic neutrino-proton cross section data available
- previous attempts (Garvey, Louis & White; Alberico et al.) to extract the strange quark axial form factor from this data were not successful; assumed dipole Q^2-dependence for G_A^s; this assumption is no longer necessary, as additional experimental information are available now (i.e. HAPPEX) and more are on the way.

E734 Results

Uncertainties shown are total (stat and sys).

Last row averages the 0.45 and 0.55 GeV^2 points together.

Q^2 (GeV)^2	$d\sigma/dQ^2(\nu p)$ (fm/GeV)^2	$d\sigma/dQ^2(\bar{\nu} p)$ (fm/GeV)^2
0.45	0.165 ± 0.033	0.0756 ± 0.0164
0.55	0.109 ± 0.017	0.0426 ± 0.0062
0.65	0.0803 ± 0.0120	0.0283 ± 0.0037
0.75	0.0657 ± 0.0098	0.0184 ± 0.0027
0.85	0.0447 ± 0.0092	0.0129 ± 0.0022
0.95	0.0294 ± 0.0074	0.0108 ± 0.0022
1.05	0.0205 ± 0.0062	0.0101 ± 0.0027
0.50	0.137 ± 0.023	0.0591 ± 0.0102

Elastic neutrino-proton cross sections

$$\frac{d\sigma}{dQ^2}(\nu p \to \nu p) = \frac{G_F^2}{2\pi}\frac{Q^2}{E_\nu^2}\left(A \pm BW + CW^2\right) \qquad \begin{matrix} + & \nu \\ \\ - & \bar{\nu} \end{matrix}$$

$$W = 4\left(E_\nu / M_p - \tau\right) \qquad\qquad \tau = Q^2 / 4M_p^2$$

$$A = \frac{1}{4}\left[\left(G_A^Z\right)^2(1+\tau) - \left(\left(F_1^Z\right)^2 - \tau\left(F_2^Z\right)^2\right)(1-\tau) + 4\tau F_1^Z F_2^Z\right]$$

$$B = -\frac{1}{4}G_A^Z\left(F_1^Z + F_2^Z\right)$$

$$C = \frac{1}{64\tau}\left[\left(G_A^Z\right)^2 + \left(F_1^Z\right)^2 + \tau\left(F_2^Z\right)^2\right]$$

Difference of Cross Sections

$$\Delta \equiv \frac{d\sigma}{dQ^2}(\nu p \to \nu p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p \to \bar{\nu} p)$$

$$= \frac{G_F^2}{2\pi}\frac{Q^2}{E_\nu^2}(2BW) = -\frac{G_F^2}{4\pi}\frac{Q^2}{E_\nu^2} G_A^Z \left(F_1^Z + F_2^Z\right) W$$

Recall $\quad F_1^Z + F_2^Z = \frac{1}{2}\left[\left(1 - 4\sin^2\theta_W\right)G_M^p - G_M^n - G_M^s\right]$

and $\quad G_A^Z = \frac{1}{2}\left(-G_A^{CC} + G_A^s\right)$

Then

$$aG_M^s - G_A^s G_M^s + bG_A^s + c = 0$$

Relates $G_M{}^s$ and $G_A{}^s$.

$a = G_A^{CC} \qquad b = \left(1 - 4\sin^2\theta_W\right)G_M^p - G_M^n \qquad c = \frac{16\pi}{W}\frac{E_\nu^2}{Q^2}\frac{\Delta}{G_F^2} - ab$

Sum of Cross Sections

$$\Sigma \equiv \frac{d\sigma}{dQ^2}(\nu p \to \nu p) + \frac{d\sigma}{dQ^2}(\bar{\nu} p \to \bar{\nu} p)$$

$$= \frac{G_F^2}{2\pi}\frac{Q^2}{E_\nu^2}\left(2A + 2CW^2\right)$$

$$= \frac{G_F^2}{4\pi}\frac{Q^2}{E_\nu^2}\left[\left(-1+\tau+\frac{W^2}{16\tau}\right)\left(F_1^Z\right)^2 + \left(+1-\tau+\frac{W^2}{16\tau}\right)\left(F_2^Z\right)^2\right.$$

$$\left. + \left(+1+\tau+\frac{W^2}{16\tau}\right)\frac{(4\pi)^2\Delta^2}{G_F^4}\frac{E_\nu^4}{Q^4}\frac{1}{W^2\left(F_1^Z+F_2^Z\right)^2} + 4\tau F_1^Z F_2^Z\right]$$

Used Δ to eliminate the dependence on the axial form factors.

F_1^Z and F_2^Z only contain electric and magnetic form factors.

This can be written as a fourth order polynomial in G_E^s and G_M^s.

Two Useful Relations

$$\Delta \equiv \frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p \rightarrow \bar{\nu} p)$$

is a function only of G_M^s and G_A^s.

$$\Sigma \equiv \frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) + \frac{d\sigma}{dQ^2}(\bar{\nu} p \rightarrow \bar{\nu} p)$$

is a function only of G_M^s and G_E^s.

Need at least one more relation to extract the three strange form factors.
That's where the PV $\vec{e}p$ measurements come in.

What you learn from the Δ expression

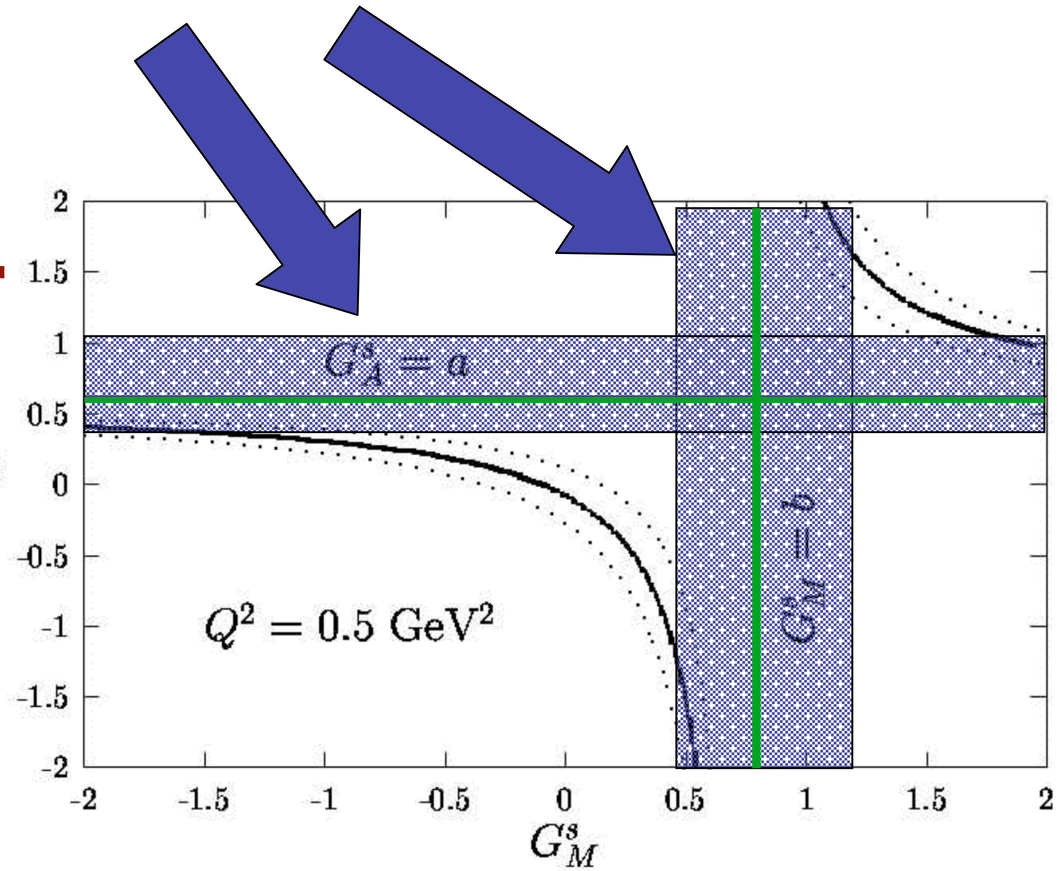
- single-valued function relating strange axial and magnetic form factors
- asymptotes correspond to the fact that if Δ is not zero, then neither G_A^Z nor $(F_1^Z + F_2^Z)$ can be zero
- **asymptotes *rule out a range of values* for the strange axial and magnetic form factors**

$$aG_M^s - G_A^s G_M^s + bG_A^s + c = 0$$

$$a = G_A^{CC}$$

$$b = \left(1 - 4\sin^2\theta_W\right)G_M^p - G_M^n$$

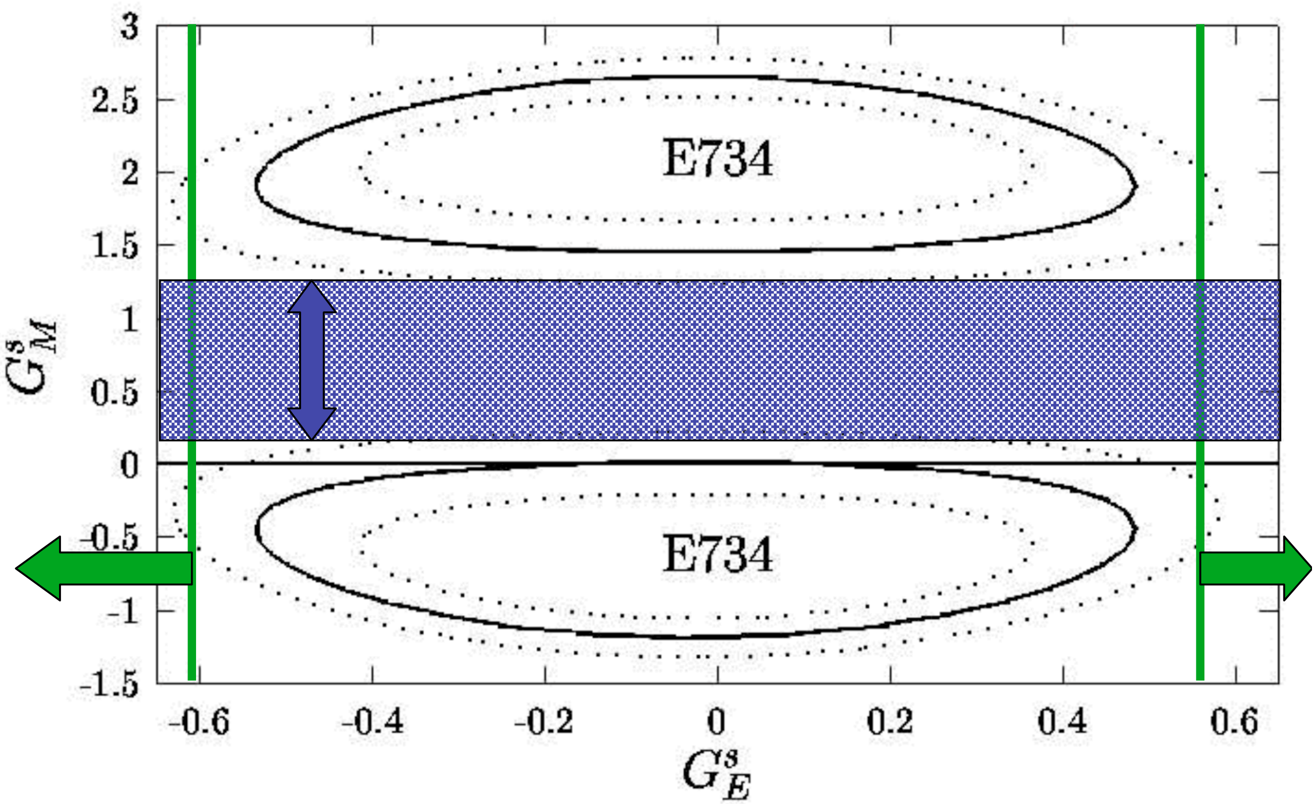
$$c = \frac{16\pi}{W}\frac{E_\nu^2}{Q^2}\frac{\Delta}{G_F^2} - ab$$



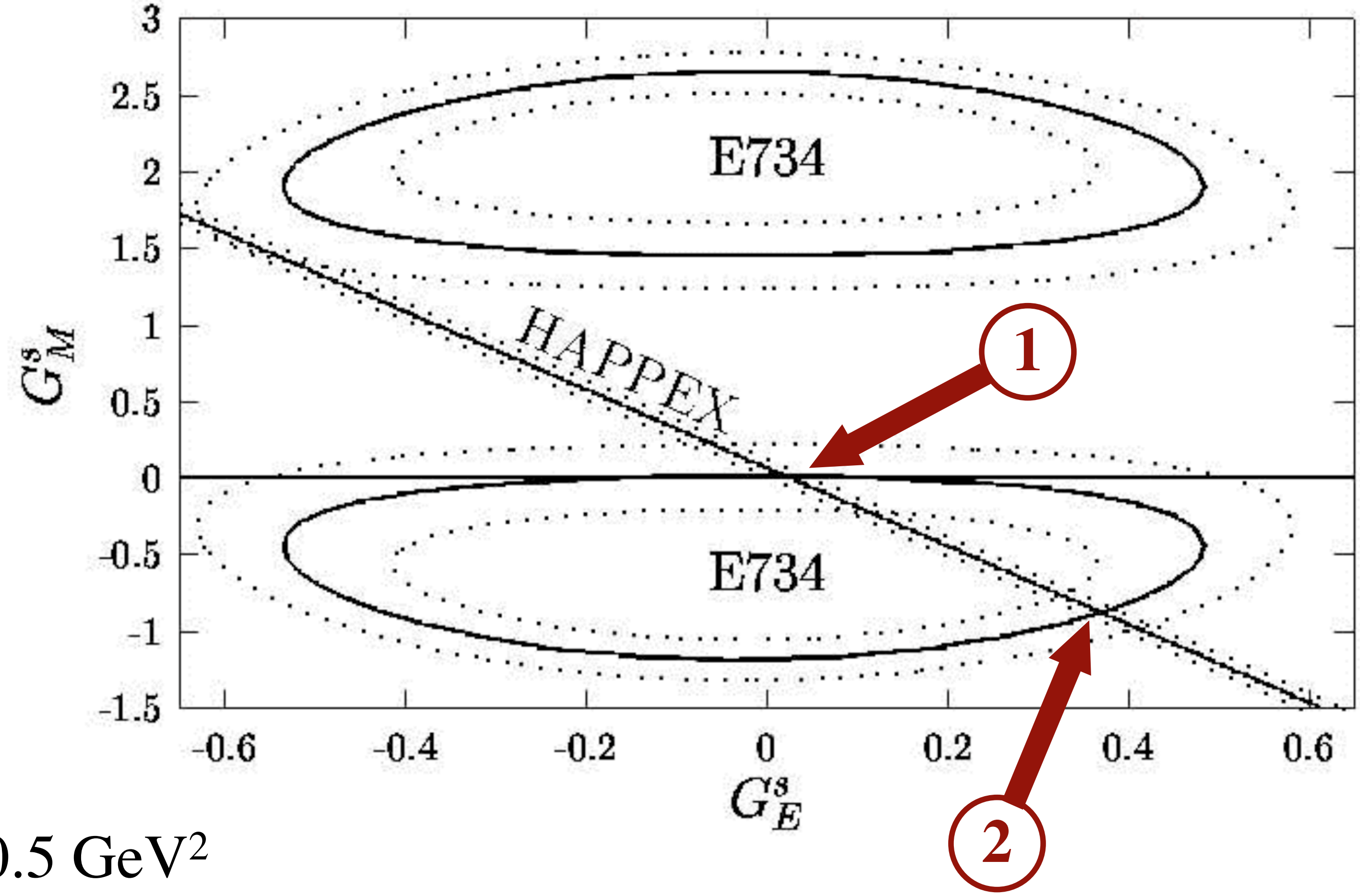
What you learn from the Σ expression

- multi-valued function of strange electric and magnetic form factors
- At this Q^2:
  - ➙ rules out $|G_E^s| > \sim 0.6$
  - ➙ rules out moderate positive values of G_M^s

$Q^2 = 0.5\ \mathrm{GeV}^2$



Combining E734 and HAPPEX

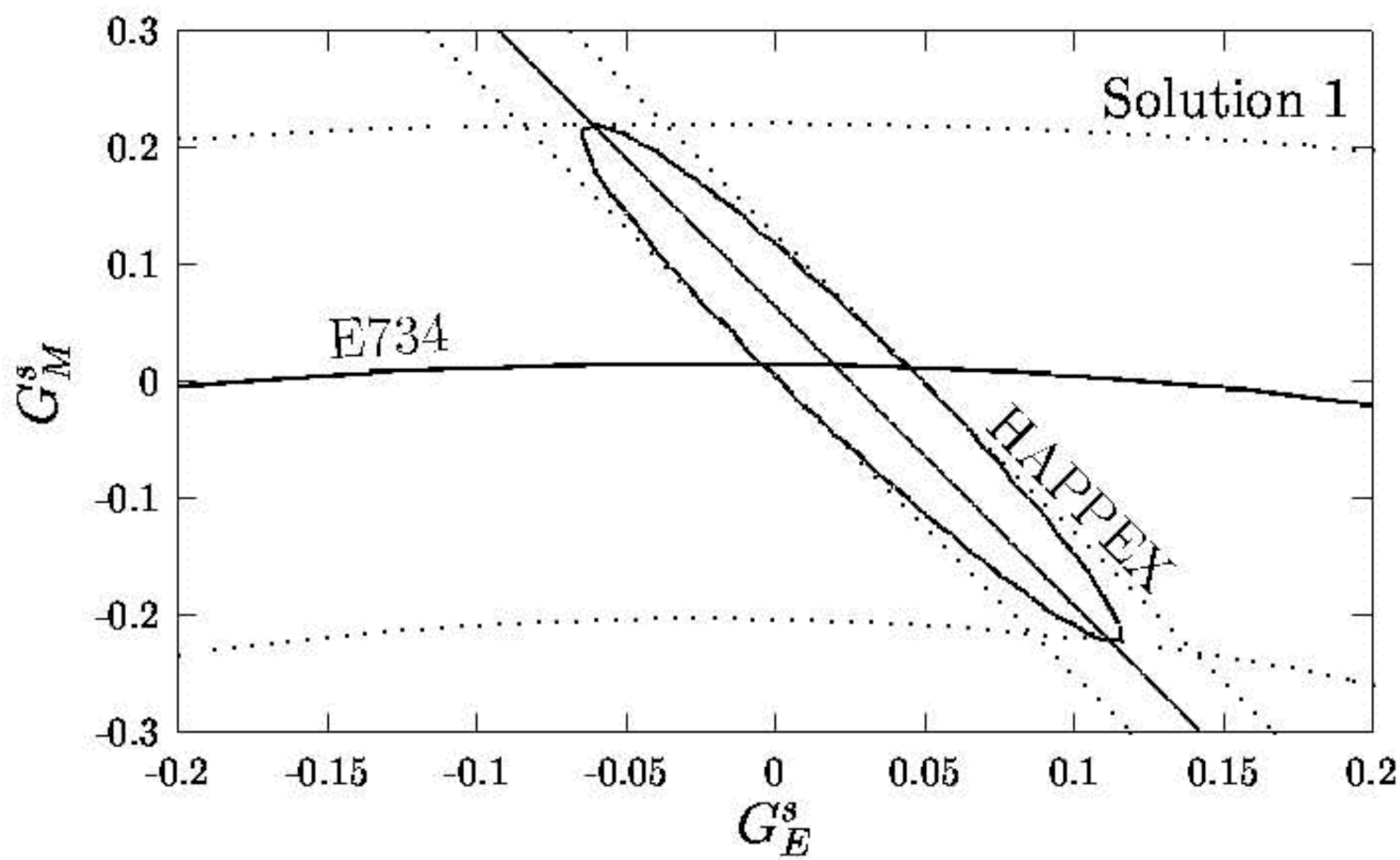


$Q^2 = 0.5\ \mathrm{GeV}^2$

There are two solutions!

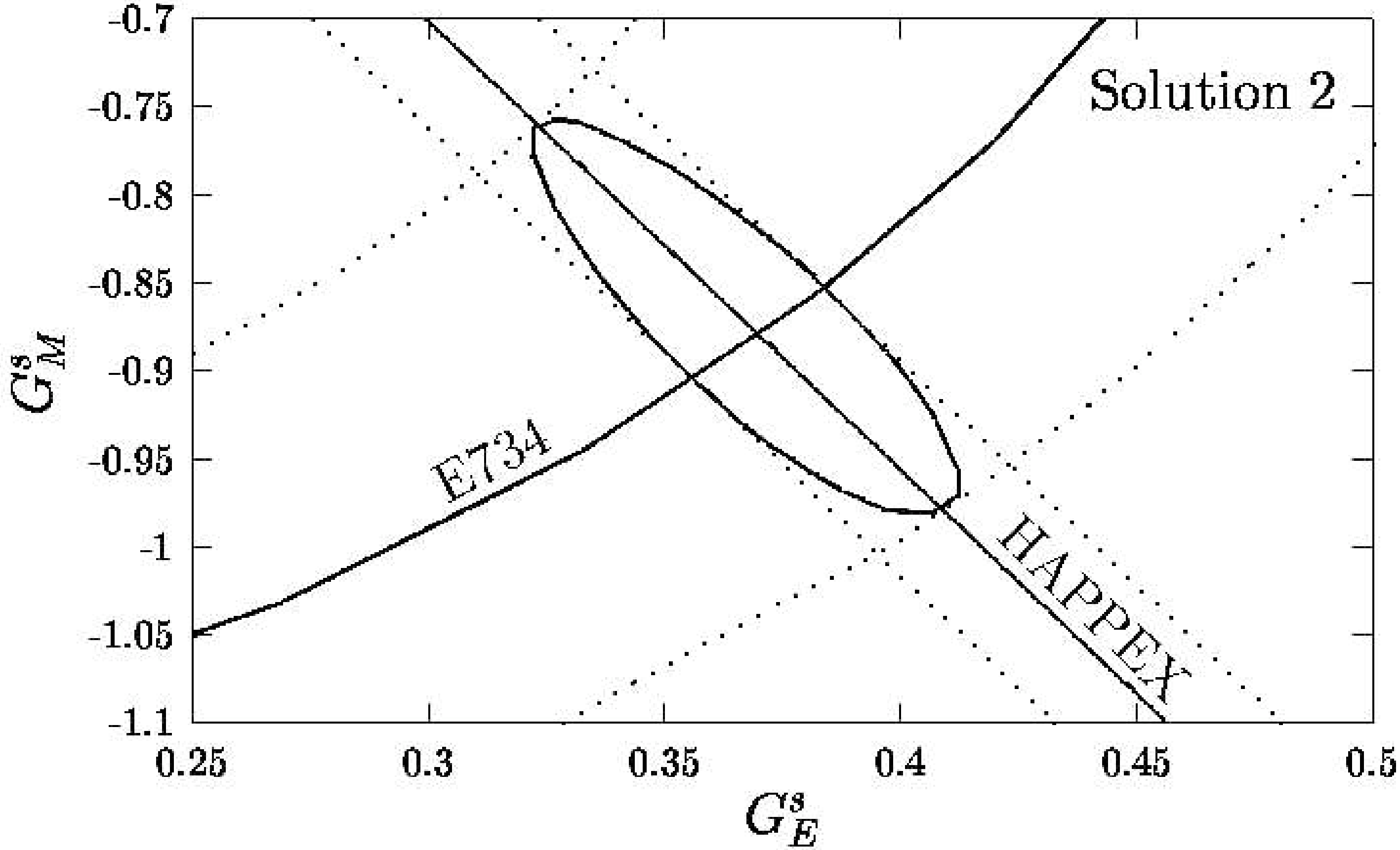
Solution 1

$G_E{}^s$	0.02±0.09
$G_M{}^s$	0.00±0.21
$G_A{}^s$	−0.09±0.05



Solution 2

$G_E^{\,s}$	0.37±0.04
$G_M^{\,s}$	–0.87±0.11
$G_A^{\,s}$	0.28±0.10



The Two Solutions

$Q^2 = 0.5$ GeV^2

	Solution 1	Solution 2
G_E^s	0.02±0.09	0.37±0.04
G_M^s	0.00±0.21	–0.87±0.11
G_A^s	–0.09±0.05	0.28±0.10

Three reasons to prefer **Solution 1**:

- G_A^s in Solution 1 is consistent with DIS estimate at $Q^2 = 0$
- G_M^s in Solution 1 is consistent with SAMPLE result at $Q^2 = 0.091$ GeV^2
- G_M^s in Solution 1 is consistent with the Lattice QCD (Leinweber et al.) prediction of $G_M^s(Q^2 = 0) = -0.051 \pm 0.021$.

Additional measurement (JLab E91-004 or G^0-Backward) needed to determine the correct solution.

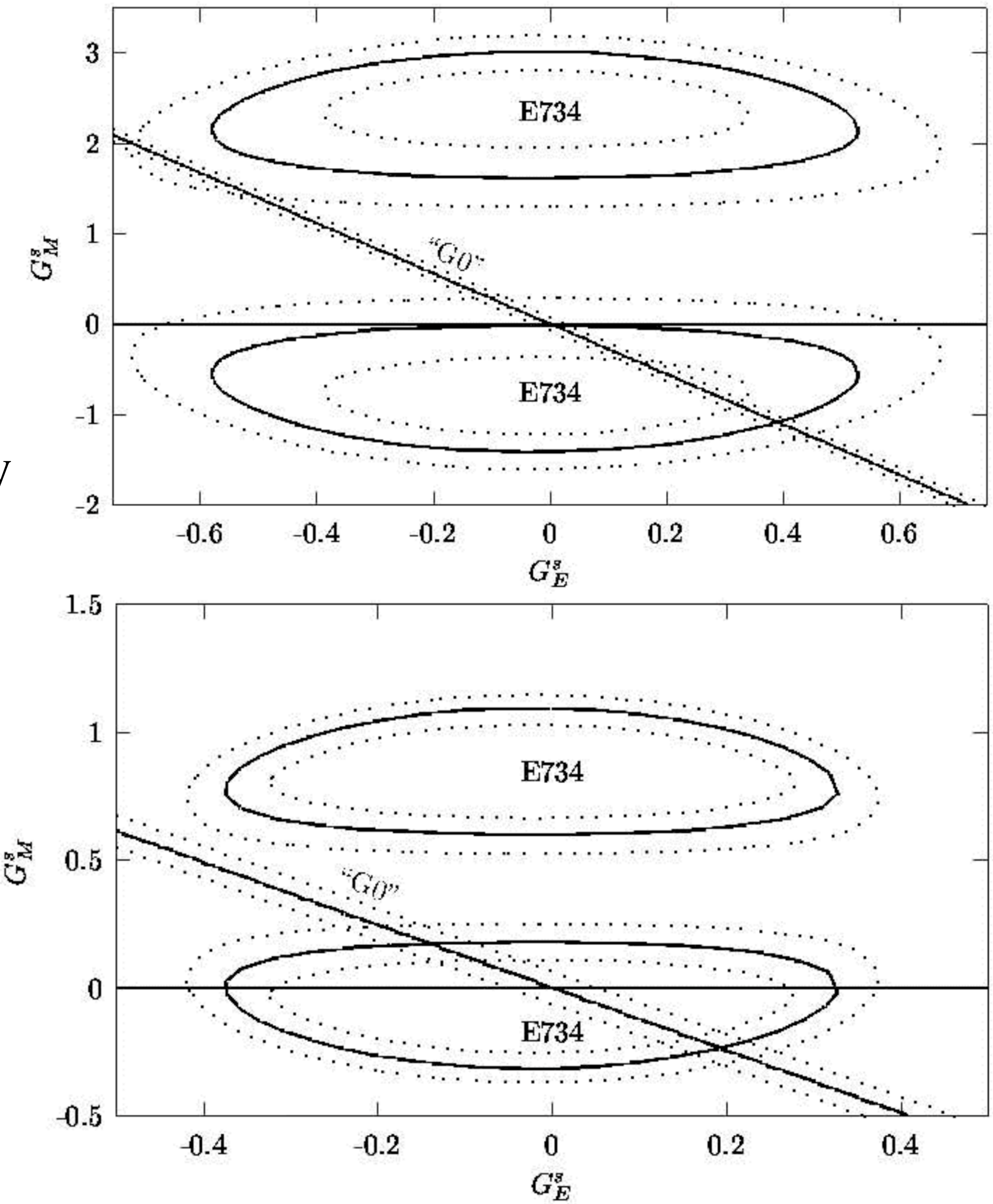
Combining E734 and G^0(Forward)

- First phase of G^0 is similar to HAPPEX: forward electron scattering, **thus little sensitivity to G_A^e**.
- Get linear combinations of G_E^s and G_M^s at four Q^2 points (0.45, 0.55, 0.75, 0.95 GeV^2) in same range as E734.
- G_A^e
  - Use a value for G_A^e from Zhu et al., OR
  - Just set $G_A^e = 0 \pm 1$ and live with slightly wider uncertainty band.
- Similar analysis as for HAPPEX will give two solutions at each Q^2 point.
- First G^0(Backward) measurement (or JLab E91-004 measurement) will select correct solution set.

What it might look like…

"Solutions" produced by forcing the G^0 "data" to go through the origin (like HAPPEX), and using $G_A^e = 0 \pm 1.$

$$G_E^s + \alpha G_M^s = 0 \pm \beta$$



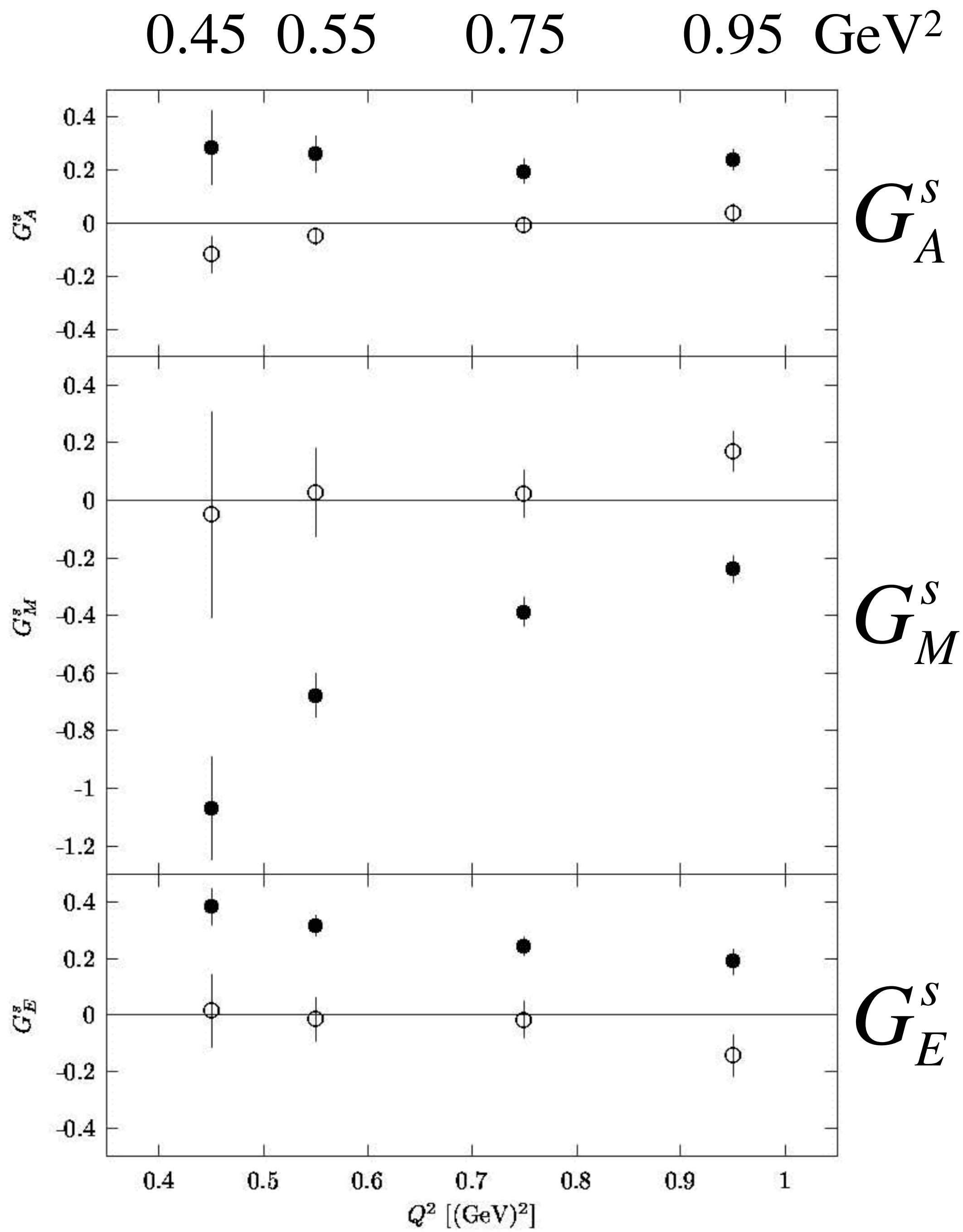
0.45 GeV^2

0.95 GeV^2

E734 + G^0(Forward) expected uncertainties

- ○ Solution 1
- • Solution 2

"Data" are clearly arbitrary, but sizes of uncertainties are not.



A future experiment to determine Δs

Even if the program I have described determines the strange axial form factor down to $Q^2 = 0.45$ GeV^2 successfully, it almost certainly will not determine the Q^2-dependence sufficiently for an extrapolation down to $Q^2 = 0$.

Also, questions remain about the normalization of the E734 data. Most of their target protons were inside of carbon nuclei, and there was not much known about nuclear transparency in the mid-1980's. The E734 collaboration **did** make a correction for transparency effects, but this issue needs to be revisited if we continue to use the E734 data. (I have the original E734 simulation code and am working on this project with a student this summer.)

A new experiment has been proposed to measure elastic and quasi-elastic neutrino-nucleon scattering to sufficiently low Q^2 to directly measure Δs.

FINeSSE

Twofold proposal:

1) Improve (with MiniBooNE) measurements of neutrino mixing phenomena

2) Measure the strange axial form factor down to $Q^2 = 0.2$ GeV^2

A Proposal for a Near Detector Experiment on the Booster Neutrino Beamline:

FINeSSE: Fermilab Intense Neutrino Scattering Scintillator Experiment

November 23, 2003

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FINeSSE Determination of Δs

Measure ratio of NC to CC neutrino scattering from nucleons:

$$R_{\mathrm{NC/CC}} = \frac{\sigma(\nu p \to \nu p)}{\sigma(\nu n \to \mu^- p)}$$

$\Rightarrow$ Numerator is sensitive to $\left(-G_A^{CC} + \boxed{G_A^s}\right)$

$\Rightarrow$ Denominator is sensitive to G_A^{CC} only

$\Rightarrow$ Both processes have unique charged particle final states signatures

$\Rightarrow$ Ratio largely eliminates uncertainties in neutrino flux, detector efficiency, and nuclear target effects

A 6% measurement of $R_{\mathrm{NC/CC}}$ down to $Q^2 = 0.2$ GeV^2 provides a ± 0.04 measurement of Δs.

In conclusion…

• Get values for all three strange form factors in the range $0.45 < Q^2 < 0.95$ GeV^2 by combining BNL E734 neutrino scattering data with JLab G^0 electron scattering data

(this work available at PRL 92 (2004) 082002 and hep-ex/0310052)

• FINeSSE the job by measuring the NC and CC neutrino scattering to sufficiently low Q^2 to get all three strange form factors over the Q^2 range $0.2 < Q^2 < 1.0$ GeV^2.

→ determine Δs