

# Don't forget to measure $\Delta s$

*or*

$\Delta s$ : There are things we know, and things we don't know...

- Programs at Bates, Mainz, and Jefferson Lab that plan to measure the **strange electric and magnetic form factors** depend on knowledge of the **strange axial form factor** -- but that form factor has never been directly measured!
- Connections between, and review of, available data linked to the strange axial form factor
- How to combine elastic  $\nu p$  and  $ep$  data to get  $G_A^s$
- Combine BNL E734  $\nu p$  data and the HAPPEX  $ep$  data to get 2 distinct solutions for the strange form factors at  $Q^2 = 0.5 \text{ GeV}^2$
- E734 and  $G^0$  and FINeSSE

## The strange axial form factor and $\Delta s$

During the early years of the “spin crisis” there was a great interest in  $\Delta s$  because it was thought to be the “solution” to the problem of the missing valence quark spin contribution to the proton spin.

By the mid 1990’s it was clear that the complete “solution” involved also the gluon and orbital angular momentum sectors, and the focus of the spin community began to shift in those directions. The inclusive polarized DIS experiments produced useful (but assumption-laden) estimates of  $\Delta s$ .

The two major projects that could have measured  $\Delta s$  directly in neutrino scattering during this time period, the BNL E734 experiment and the LANL LSND experiment, failed to do so conclusively.

In the meantime, a program of parity-violating  $eN$  experiments arose, with the goal of measuring the strange *electromagnetic* form factors of the nucleon. This cannot be done without knowing the strange *axial* form factor too, but since  $\Delta s$  had been “measured” in polarized DIS they wisely proceeded with their plans.

# Data on $\Delta s$ from Inclusive Polarized Deep Inelastic Scattering

In the quark-parton model, inclusive scattering of leptons from nucleon targets measures the nucleon structure function  $F_1$ :

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x) \quad q(x) = \text{p.d.f.}$$

Inclusive scattering of **polarized** leptons from **polarized** nucleon targets measures the **spin-dependent** nucleon structure function  $g_1$ :

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x) \quad \Delta q(x) = \text{polarized p.d.f.}$$

The first moment of the polarized p.d.f. is the contribution of that flavor to the nucleon spin:

$$\Delta q \equiv \int_0^1 \Delta q(x) dx$$

The  $\Delta q$  are also called the “axial charges” because they are related to the matrix elements of the axial current:

$$\Delta q \propto \bar{q} \gamma_\mu \gamma_5 q$$

## QCD: $Q^2$ -dependence and radiative corrections

In leading order QCD, these functions take on a scale dependence:

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q(x, Q^2)$$

In NLO QCD, there are significant radiative corrections and the relation between  $g_1$  and the  $\Delta q$  is more complex.

The discussion here will be limited to the leading-order QCD analysis for two reasons:

- Including the NLO terms does not change the answer for  $\Delta s$  very much.
- The problems I will point out exist at all orders, because they are problems coming from the data itself.

# The classic LO QCD analysis of inclusive polarized DIS data

[SMC: Adeva *et al.*, Phys. Lett. B 412 (1997) 414]

1) Take measured values of  $g_1(x, Q^2)$ , covering ranges of  $x$  and  $Q^2$  of  $0.003 < x < 0.70$  and  $1.3 < Q^2 < 58.0$ , and use QCD to evolve all data to a common value of  $Q^2 = 10 \text{ GeV}^2$ . The evolution procedure leads to a fit function for  $g_1$  in the measured  $x$ -region.

2) Extrapolate the measured values to  $x = 1$  assuming a constant value of the experimental asymmetry. (This step contributes little to the **area** of  $g_1$  and also very little to the uncertainty and I don't discuss it again.)

3) Extrapolate the measured values to  $x = 0$ . It is unclear how to do this, so two approaches are used\*. One is to assume  $g_1 = \text{constant}$  for  $x < 0.003$  --- this is called the "Regge assumption" for not very good reasons. The other approach is to simply use the QCD fit from step 1, extended to  $x = 0$ .

\*Dogs walk around in a circle twice before laying down....

## Completing the analysis, using both fits:

4) Integrate  $g_1$  over  $0 < x < 1$ :  $\Gamma_1 = \int_0^1 g_1(x) dx = \begin{cases} 0.142 \pm 0.017 & \text{"Regge"} \\ 0.130 \pm 0.017 & \text{QCD fit} \end{cases}$

This integral relates the three axial charges:

$$\Gamma_1 = \int_0^1 g_1(x) dx = \frac{1}{2} \sum_q e_q^2 \int_0^1 \Delta q(x) dx = \frac{1}{2} \left[ \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right]$$

5) Assuming  $SU(3)_f$  is a valid symmetry of the baryon octet, and using hyperon beta decay data, we can determine two other relations between the three axial charges:

$$\Delta u - \Delta d = \frac{g_A}{g_V} = F + D \quad \text{and} \quad \Delta u + \Delta d - 2\Delta s = 3F - D$$

where  $\frac{g_A}{g_V} = 1.2601 \pm 0.0025$  and  $\frac{F}{D} = 0.575 \pm 0.016$  [in 1997]

6) Solve for the axial charges!

	"Regge"	QCD fit
$\Delta u$	$0.84 \pm 0.06$	$0.80 \pm 0.06$
$\Delta d$	$-0.42 \pm 0.06$	$-0.46 \pm 0.06$
$\Delta s$	$-0.08 \pm 0.06$	$-0.12 \pm 0.06$

## Data on $\Delta s$ from Semi-Inclusive Polarized Deep Inelastic Scattering

In semi-inclusive DIS, a leading hadron is observed in coincidence with the scattered lepton. This allows a statistical identification of the struck quark, and hence a measurement of the  $x$ -dependence of the individual  $\Delta q(x)$  distribution functions. (Inclusive scattering only measures the total structure function  $g_1(x)$ .)

The HERMES experiment on the HERA ring at DESY was especially designed to make this measurement.

HERMES measured double-spin asymmetries in the production of charged hadrons in polarized deep-inelastic scattering of positrons from polarized targets: Specifically, the asymmetry in the production of charged pions on targets of hydrogen and deuterium, and of charged kaons in scattering from deuterium.

## HERMES measurement of $\Delta q(x)$

There is no assumption of  $SU(3)_f$  symmetry in their analysis. They extract the following quark polarization distributions, over the range  $0.023 < x < 0.30$  :

$$\frac{\Delta u}{u}(x) \quad \frac{\Delta d}{d}(x) \quad \frac{\Delta \bar{u}}{\bar{u}}(x) \quad \frac{\Delta \bar{d}}{\bar{d}}(x) \quad \frac{\Delta s}{s}(x)$$

where  $\frac{\Delta s}{s}(x)$  is defined to be the sum of  $\frac{\Delta s}{s}(x)$  and  $\frac{\Delta \bar{s}}{\bar{s}}(x)$ .

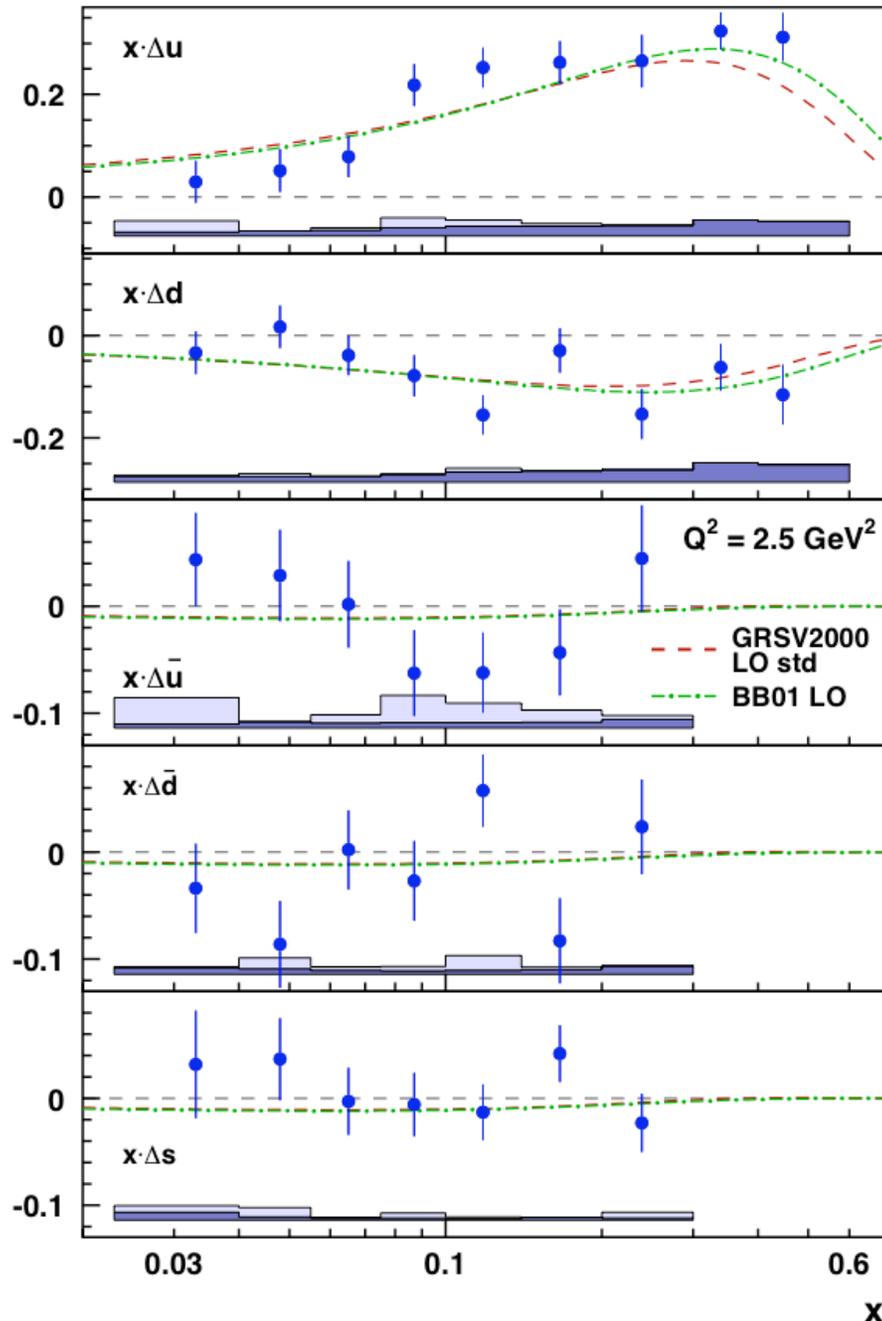
They integrate the strange distribution to obtain:

$$" \Delta s " = \int_{x=0.023}^{0.30} \Delta s(x) dx = +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$$

[This would only be the true  $\Delta s$  if the integral was over  $0 \leq x \leq 1$ .]

# HERMES Results: quark helicity distributions

PRL 92 (2004) 012005



$u$  and  $d$  quarks  
polarized

$$Q^2 = 2.5 \text{ GeV}^2$$

$$0.023 < x < 0.30$$

sea quarks  
unpolarized

$$" \Delta s " = \int_{x=0.023}^{0.30} \Delta s(x) dx$$

$$= +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$$

# So, where did the negative $\Delta s$ go?

The HERMES article [PRL 92 (2004) 012005] contains this very interesting remark:

“The strange sea distribution was previously found to be negatively polarized in the analysis of only inclusive data assuming SU(3) symmetry applied to hyperon beta decay data. However, the first moments from such analyses evaluated over the measured  $x$  range  $(\Delta s + \Delta \bar{s})/2 \equiv \int_{0.023}^{0.3} \Delta s(x) dx$  are typically not in disagreement with the partial moment of the density extracted here:  $\Delta s = +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{sys})$ .”

**Translation: If both analyses are correct, then all of the negative contribution to  $\Delta s$  from the SU(3)<sub>f</sub> analysis of inclusive DIS data came from lower values of  $x$ , that is from  $x < 0.023$ .**

# Low- $x$ strange quark polarization

The HERMES result suggests that the strange quark helicity distribution  $\Delta s(x) \sim 0$  for  $x > 0.023$ .

At the same time, we have our result from the  $SU(3)_f$  analysis of hyperon beta decay and inclusive polarized DIS data, including an extrapolation to  $x = 0$ :

$$\Delta s = \int_0^1 \Delta s(x) dx \approx -0.10 \pm 0.06$$

If these two results are both true, then the average value of  $\Delta s(x)$  in the range  $x < 0.023$  must be  $\sim -5$ . That's not impossible, as  $s(x)$  is  $\sim 20-300$  in the range  $x \sim 10^{-2}$  to  $10^{-3}$  (CTEQ6). But we would need a mechanism that would “turn on” the strange quark polarization suddenly at these low  $x$  values.

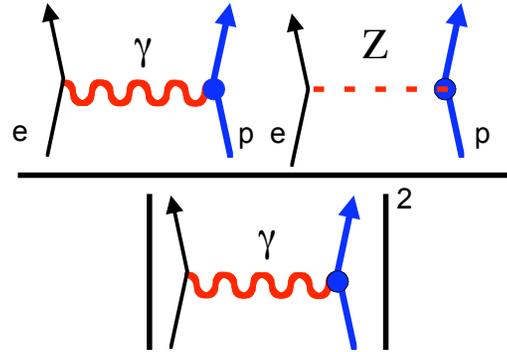
# Other explanations and considerations

- $SU(3)_f$  symmetry is not valid. A few years ago, when experimental data was sparse, we needed  $SU(3)_f$  symmetry to extend the reach of the analysis. But with so much data available now, we don't need to use  $SU(3)_f$  as a crutch anymore.
- The extrapolations to  $x = 0$  are not valid. There is no way to prove or disprove this except via additional experimental measurements.

A direct measurement of  $\Delta s$  would certainly serve to clarify these issues!

# Parity Violating Electron Scattering

polarized electrons  
unpolarized target



$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

for a nucleon:

$$= \left[ \frac{-G_F Q^2}{4\pi\alpha\sqrt{2}} \right] \frac{\varepsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\sin^2\theta_W)\varepsilon' G_M^\gamma G_A^e}{\varepsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

$$\tau = \frac{Q^2}{4M^2}$$

$$\varepsilon = \left[ 1 + 2(1 + \tau)\tan^2(\theta/2) \right]^{-1}$$

$$\varepsilon' = \sqrt{(1 - \varepsilon^2)\tau(1 + \tau)}$$

forward angles

HAPPEX, Mainz,  $G^0$ : sensitive to

$G_E^S$  and  $G_M^S$

backward angles

SAMPLE,  $G^0$ : sensitive to

$G_M^S$  and  $G_A^e$

effective axial-vector  
e-N form factor

# The effective axial form factor seen in parity-violating $\vec{e}N$ scattering

$$G_A^e = -G_A^{CC} + G_A^s + G_A^{\text{ewm}}$$

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$$G_A^{CC} = \frac{g_A}{\left(1 + Q^2 / M_A^2\right)^2} \quad g_A = 1.267 \quad \text{Charged-current axial form factor; dipole } Q^2 \text{ behavior}$$
$$M_A = 1.026 \text{ GeV}$$

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$$G_A^s(Q^2 = 0) = \Delta s = -0.10 \pm 0.06$$

[estimate from DIS data]

Strange quark axial form factor; unknown  $Q^2$  behavior

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$G_A^{\text{ewm}}$  Contains **electroweak mixing** contributions ---- must be calculated...

# Isoscalar and isovector components of $G_A^e$

The terms in  $G_A^{\text{ewm}}$ , being corrections, are either additive terms or are proportional to  $G_A^{\text{CC}}$  and  $G_A^s$ . It is customary to rewrite  $G_A^e = -G_A^{\text{CC}} + G_A^s + G_A^{\text{ewm}}$  in terms of isoscalar and isovector components :

$$G_A^e = G_A^e(T=0) + G_A^e(T=1)$$

with  $G_A^e(T=0) = G_A^s + R_A^{T=0}$

and  $G_A^e(T=1) = -G_A^{\text{CC}}(1 + R_A^{T=1})$

The correction factors  $R_A^{T=0}$  and  $R_A^{T=1}$  have been calculated at  $Q^2 = 0$  in the context of heavy - baryon chiral perturbation theory by Zhu et al., Phys. Rev. D62 (2000) 033008.

$$R_A^{T=0} = 0.03 \pm 0.05 \quad R_A^{T=1} = -0.23 \pm 0.24$$

# The SAMPLE Result

SAMPLE measured PV asymmetry in backward angle elastic  $\bar{e}N$  scattering at  $Q^2 = 0.091 \text{ GeV}^2$ , using both hydrogen and deuterium targets :

$$A_H = -5.61 \pm 0.67 \pm 0.88 \text{ ppm} \quad (\text{Phys. Lett. B583 (2004) 79})$$

$$A_D = -7.77 \pm 0.73 \pm 0.62 \text{ ppm} \quad (\text{Phys. Rev. Lett. 92 (2004) 102003})$$

Assuming:  $G_A^s \cong \Delta s = -0.1 \pm 0.1$  (E. Beise, priv. comm.),

$G_E^s = 0$  (this must be nearly zero near  $Q^2 = 0$ ), and

the correction factor  $R_A^{T=0} = 0.03 \pm 0.05$  (Zhu et al.),

then these two asymmetries are related to  $G_M^s$  and  $G_A^e(T=1)$  as follows :

$$A_H = -5.56 + 3.37G_M^s + 1.54G_A^e(T=1) \text{ ppm}$$

$$A_D = -7.06 + 0.77G_M^s + 1.66G_A^e(T=1) \text{ ppm}$$

SAMPLE reports a measurement of  $G_M^s$  and  $G_A^e(T=1)$ :

$$G_M^s = 0.23 \pm 0.36 \pm 0.40 \quad G_A^e(T=1) = -0.53 \pm 0.57 \pm 0.50 \quad (\text{E. Beise, priv. comm.})$$

The reason for separating out the  $G_A^e(T=1)$  term was to test the more difficult aspect of the

Zhu et al. calculation, namely the isovector correction factor:  $R_A^{T=1} = -0.23 \pm 0.24$ . Using this value, Zhu et al. are able to predict  $G_A^e(T=1) = -0.83 \pm 0.26$ , in agreement with the experiment.

## Making use of the theory...

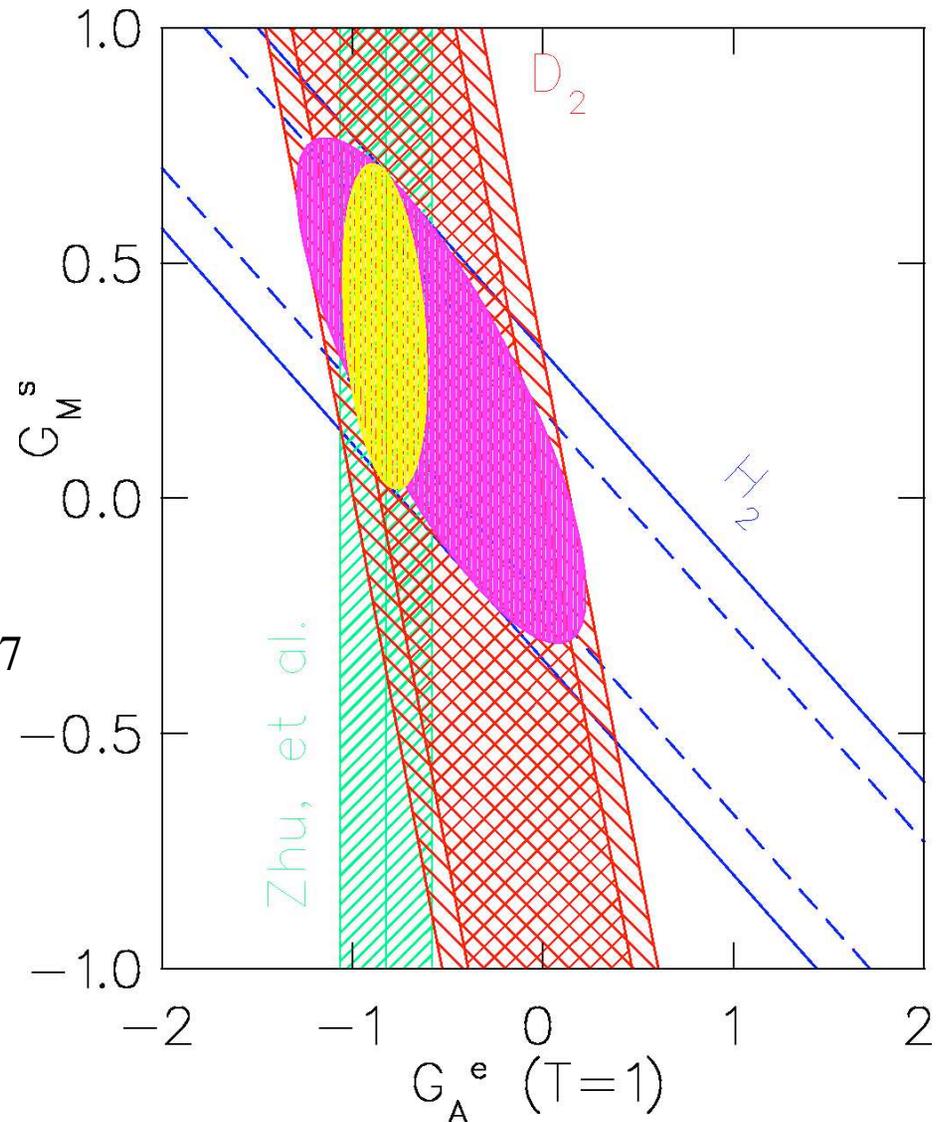
If one believes the calculation of Zhu et al., one may use instead their value of  $G_A^e(T=1)$  with the experimental value of  $A_H$  and extract a value for  $G_M^s$  :

$$G_M^s(Q^2 = 0.1 \text{ GeV}^2) = 0.37 \pm 0.20 \pm 0.26 \pm 0.07$$

[SAMPLE : Phys. Lett. B583 (2004) 79]

But if you believe the calculation of the correction factors, why not discard the assumption of  $G_A^s = -0.1 \pm 0.1$ ?

Instead, why not use the forward - angle hydrogen data of PVA4 and the backward - angle data of SAMPLE to extract  $G_A^s$  along with  $G_M^s$  ?



## Combining SAMPLE and PVA4 at $Q^2 = 0.1 \text{ GeV}^2$

Write the hydrogen asymmetries as linear combinations of  $G_E^s$  and  $G_M^s$  and  $G_A^s$  :

$$A^P = A_0^P + A_E^P G_E^s + A_M^P G_M^s + A_A^P G_A^s$$

$$\text{where } A_0^P = -\frac{G_F Q^2}{4\pi\sqrt{2}\alpha} \frac{1}{\varepsilon G_E^{p^2} + \tau G_M^{p^2}} \left\{ \begin{array}{l} \varepsilon G_E^p \left[ (1 - 4 \sin^2 \theta_W)(1 + R_V^p) G_E^p - (1 + R_V^n) G_E^n \right] \\ \tau G_M^p \left[ (1 - 4 \sin^2 \theta_W)(1 + R_V^p) G_M^p - (1 + R_V^n) G_M^n \right] \\ -\varepsilon' G_M^p (1 - 4 \sin^2 \theta_W) \left[ -(1 + R_A^{T=1}) G_A^{CC} + R_A^{T=0} G_A^8 \right] \end{array} \right\}$$

$$A_E^P = \frac{G_F Q^2}{4\pi\sqrt{2}\alpha} \frac{1}{\varepsilon G_E^{p^2} + \tau G_M^{p^2}} \left\{ \varepsilon G_E^p (1 + R_V^0) \right\}$$

$$A_M^P = \frac{G_F Q^2}{4\pi\sqrt{2}\alpha} \frac{1}{\varepsilon G_E^{p^2} + \tau G_M^{p^2}} \left\{ \tau G_M^p (1 + R_V^0) \right\}$$

$$A_A^P = \frac{G_F Q^2}{4\pi\sqrt{2}\alpha} \frac{1}{\varepsilon G_E^{p^2} + \tau G_M^{p^2}} \left\{ \varepsilon' G_M^p (1 - 4 \sin^2 \theta_W) (1 + R_A^0) \right\}$$

Then

$$A_{\text{PVA4}}^P = -1.77 + 9.24 G_E^s + 0.883 G_M^s + 0.226 G_A^s \text{ ppm} \quad (Q^2 = 0.1 \text{ GeV}^2, \theta_e = 35^\circ)$$

$$A_{\text{SAMPLE}}^P = -6.85 + 2.02 G_E^s + 3.47 G_M^s + 1.59 G_A^s \text{ ppm} \quad (Q^2 = 0.1 \text{ GeV}^2, \theta_e = 145^\circ)$$

$$A_{\text{PVA4}}^P = -1.77 + 9.24G_E^s + 0.883G_M^s + 0.226G_A^s \text{ ppm} \quad (Q^2 = 0.1 \text{ GeV}^2, \theta_e = 35^\circ)$$

$$A_{\text{SAMPLE}}^P = -6.85 + 2.02G_E^s + 3.47G_M^s + 1.59G_A^s \text{ ppm} \quad (Q^2 = 0.1 \text{ GeV}^2, \theta_e = 145^\circ)$$

I'll continue to assume that the strange electric form factor is zero at this low  $Q^2$ , but note that even a small value would play a role!

  $A_{\text{PVA4}}^P = -1.77 + 0.883G_M^s + 0.226G_A^s \text{ ppm} = -1.40 \pm 0.40$

$$A_{\text{SAMPLE}}^P = -6.85 + 3.47G_M^s + 1.59G_A^s \text{ ppm} = -5.61 \pm 1.11$$

Note that the determinant of this linear system is small:

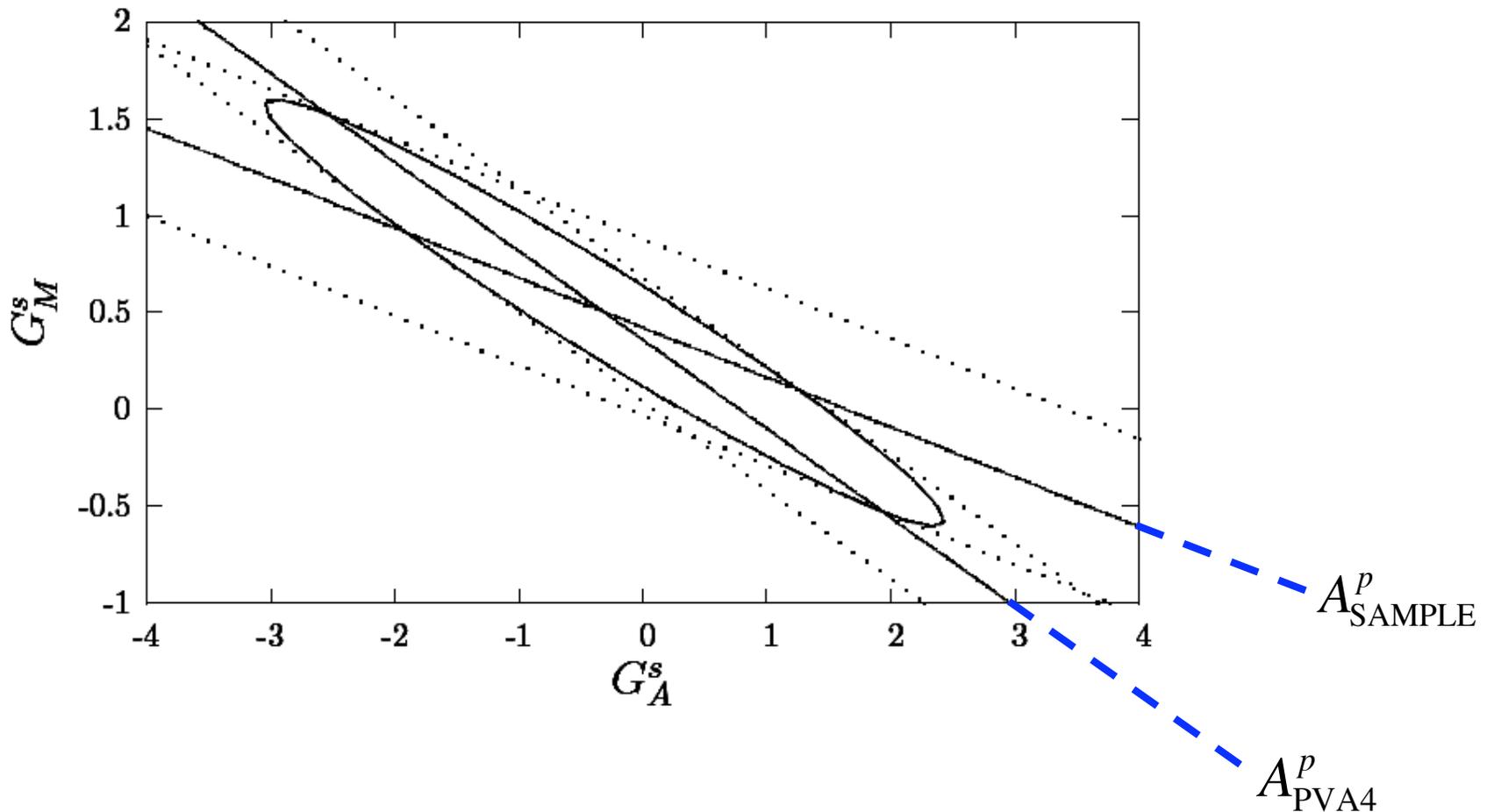
$$0.883 \times 1.59 - 0.226 \times 3.47 = 0.620$$

Therefore this is not a good way to find  $G_M^s$  and  $G_A^s$ .

Note also that  $A_A^P < A_M^P$  in both expressions.

Therefore this is a really **bad** way to find  $G_A^s$ .

At any rate, the answer is  $G_M^s = 0.4 \pm 1.0$      $G_A^s = -0.2 \pm 2.6$   
 $Q^2 = 0.1 \text{ GeV}^2$



Can't measure  $\Delta s$  in PV  $eN$  experiments.

# The HAPPEX Result

HAPPEX measured forward PV  $\vec{e}N$  asymmetry at  $Q^2 = 0.477 \text{ GeV}^2$ .

Their result is a linear combination of  $G_E^s$  and  $G_M^s$  :

$$G_E^s + 0.392G_M^s = 0.025 \pm 0.020 \pm 0.014$$

This contains a contribution from  $G_A^e$  as well, so there may be some additional uncertainty in the result.

But their sensitivity to  $G_A^e$  is about 4% of that to  $G_E^s$  and  $G_M^s$ , because the measurement is at a very forward electron scattering angle. (No axial contribution at  $\theta_e = 0$ .) So even a large uncertainty in  $G_A^e$  at this  $Q^2$  would not pose a significant problem in the interpretation of this result.

Forward-angle PV  $eN$  asymmetry measurements establish a relationship between the strange electric and magnetic form factors.

# The BNL E734 Experiment

- performed in mid-1980's
- measured neutrino- and antineutrino-proton elastic scattering
- used wide band neutrino and anti-neutrino beams of  $\langle E_\nu \rangle = 1.25$  GeV
- covered the range  $0.45 < Q^2 < 1.05$  GeV<sup>2</sup>
- large liquid-scintillator target-detector system
- still the only elastic neutrino-proton cross section data available
- previous attempts (Garvey, Louis & White; Alberico et al.) to extract the strange quark axial form factor from this data were not successful; assumed dipole  $Q^2$ -dependence for  $G_A^s$ ; this assumption is no longer necessary, as additional experimental information are available now (i.e. HAPPEX) and more are on the way.

# E734 Results

Uncertainties shown are total (stat and sys).  
Last row averages the 0.45 and 0.55 GeV<sup>2</sup> points together.

$Q^2$ (GeV) <sup>2</sup>	$d\sigma/dQ^2(\nu p)$ (fm/GeV) <sup>2</sup>	$d\sigma/dQ^2(\bar{\nu} p)$ (fm/GeV) <sup>2</sup>
0.45	0.165 ± 0.033	0.0756 ± 0.0164
0.55	0.109 ± 0.017	0.0426 ± 0.0062
0.65	0.0803 ± 0.0120	0.0283 ± 0.0037
0.75	0.0657 ± 0.0098	0.0184 ± 0.0027
0.85	0.0447 ± 0.0092	0.0129 ± 0.0022
0.95	0.0294 ± 0.0074	0.0108 ± 0.0022
1.05	0.0205 ± 0.0062	0.0101 ± 0.0027
0.50	0.137 ± 0.023	0.0591 ± 0.0102

# Elastic neutrino-proton cross sections

$$\frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) = \frac{G_F^2}{2\pi} \frac{Q^2}{E_\nu^2} (A \pm BW + CW^2) \quad \begin{array}{l} + \nu \\ - \bar{\nu} \end{array}$$

$$W = 4(E_\nu/M_p - \tau) \quad \tau = Q^2/4M_p^2$$

$$A = \frac{1}{4} \left[ (G_A^Z)^2 (1 + \tau) - \left( (F_1^Z)^2 - \tau (F_2^Z)^2 \right) (1 - \tau) + 4\tau F_1^Z F_2^Z \right]$$

$$B = -\frac{1}{4} G_A^Z (F_1^Z + F_2^Z)$$

$$C = \frac{1}{64\tau} \left[ (G_A^Z)^2 + (F_1^Z)^2 + \tau (F_2^Z)^2 \right]$$

# Difference of Cross Sections

$$\Delta \equiv \frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p \rightarrow \bar{\nu} p)$$

$$= \frac{G_F^2}{2\pi} \frac{Q^2}{E_\nu^2} (2BW) = - \frac{G_F^2}{4\pi} \frac{Q^2}{E_\nu^2} G_A^Z (F_1^Z + F_2^Z) W$$

Recall  $F_1^Z + F_2^Z = \frac{1}{2} \left[ (1 - 4 \sin^2 \theta_W) G_M^p - G_M^n - G_M^s \right]$

and  $G_A^Z = \frac{1}{2} (-G_A^{CC} + G_A^s)$

Then

$$aG_M^s - G_A^s G_M^s + bG_A^s + c = 0$$

Relates  $G_M^s$  and  $G_A^s$ .

$$a = G_A^{CC} \quad b = (1 - 4 \sin^2 \theta_W) G_M^p - G_M^n \quad c = \frac{16\pi}{W} \frac{E_\nu^2}{Q^2} \frac{\Delta}{G_F^2} - ab$$

# Sum of Cross Sections

$$\begin{aligned}
 \Sigma &\equiv \frac{d\sigma}{dQ^2}(vp \rightarrow vp) + \frac{d\sigma}{dQ^2}(\bar{\nu}p \rightarrow \bar{\nu}p) \\
 &= \frac{G_F^2}{2\pi} \frac{Q^2}{E_\nu^2} (2A + 2CW^2) \\
 &= \frac{G_F^2}{4\pi} \frac{Q^2}{E_\nu^2} \left[ \left( -1 + \tau + \frac{W^2}{16\tau} \right) (F_1^Z)^2 + \left( +1 - \tau + \frac{W^2}{16\tau} \right) (F_2^Z)^2 \right. \\
 &\quad \left. + \left( +1 + \tau + \frac{W^2}{16\tau} \right) \frac{(4\pi)^2 \Delta^2}{G_F^4} \frac{E_\nu^4}{Q^4} \frac{1}{W^2 (F_1^Z + F_2^Z)^2} + 4\tau F_1^Z F_2^Z \right]
 \end{aligned}$$

Used  $\Delta$  to eliminate the dependence on the axial form factors.

$F_1^Z$  and  $F_2^Z$  only contain electric and magnetic form factors.

This can be written as a fourth order polynomial in  $G_E^S$  and  $G_M^S$ .

## Two Useful Relations

$$\Delta \equiv \frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p \rightarrow \bar{\nu} p)$$

is a function only of  $G_M^s$  and  $G_A^s$ .

$$\Sigma \equiv \frac{d\sigma}{dQ^2}(\nu p \rightarrow \nu p) + \frac{d\sigma}{dQ^2}(\bar{\nu} p \rightarrow \bar{\nu} p)$$

is a function only of  $G_M^s$  and  $G_E^s$ .

Need at least one more relation to extract the three strange form factors.

That's where the PV  $\vec{e}p$  measurements come in.

# What you learn from the $\Delta$ expression

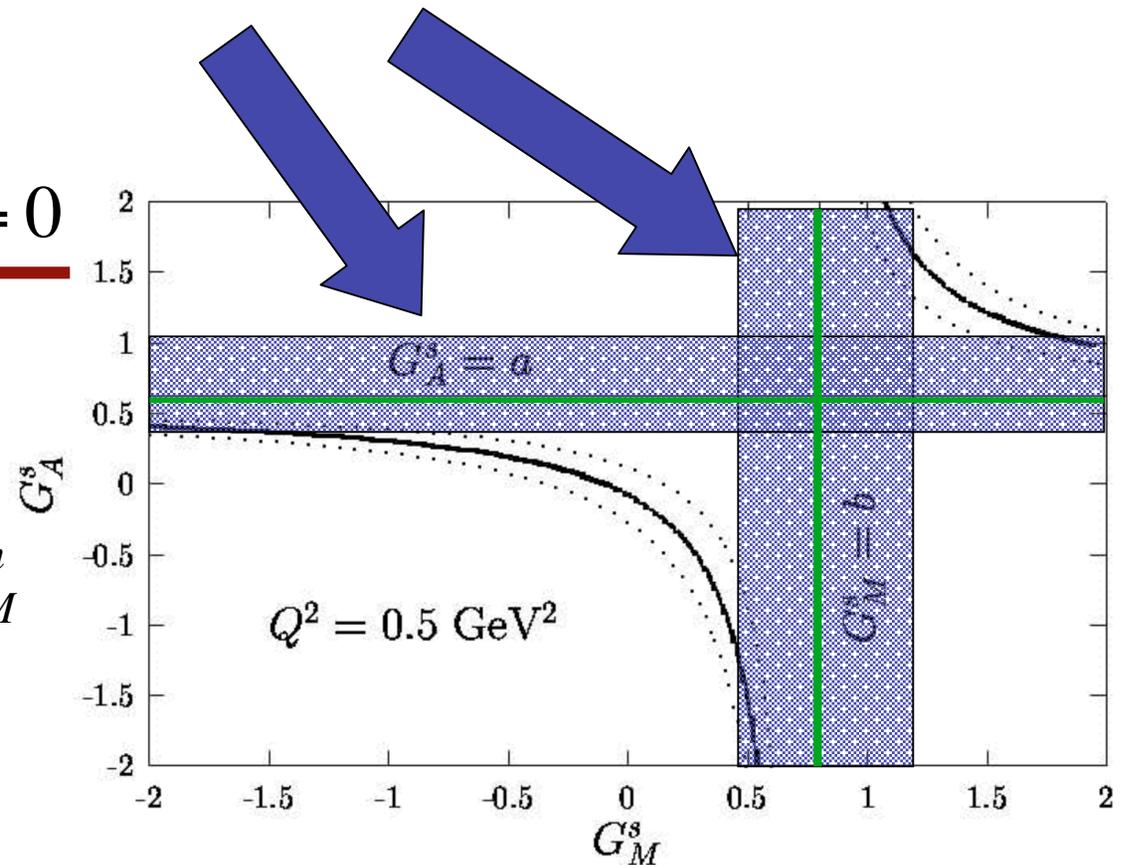
- single-valued function relating strange axial and magnetic form factors
- asymptotes correspond to the fact that if  $\Delta$  is not zero, then neither  $G_A^Z$  nor  $(F_1^Z + F_2^Z)$  can be zero
- asymptotes rule out a range of values for the strange axial and magnetic form factors

$$aG_M^s - G_A^s G_M^s + bG_A^s + c = 0$$

$$a = G_A^{CC}$$

$$b = (1 - 4 \sin^2 \theta_W) G_M^p - G_M^n$$

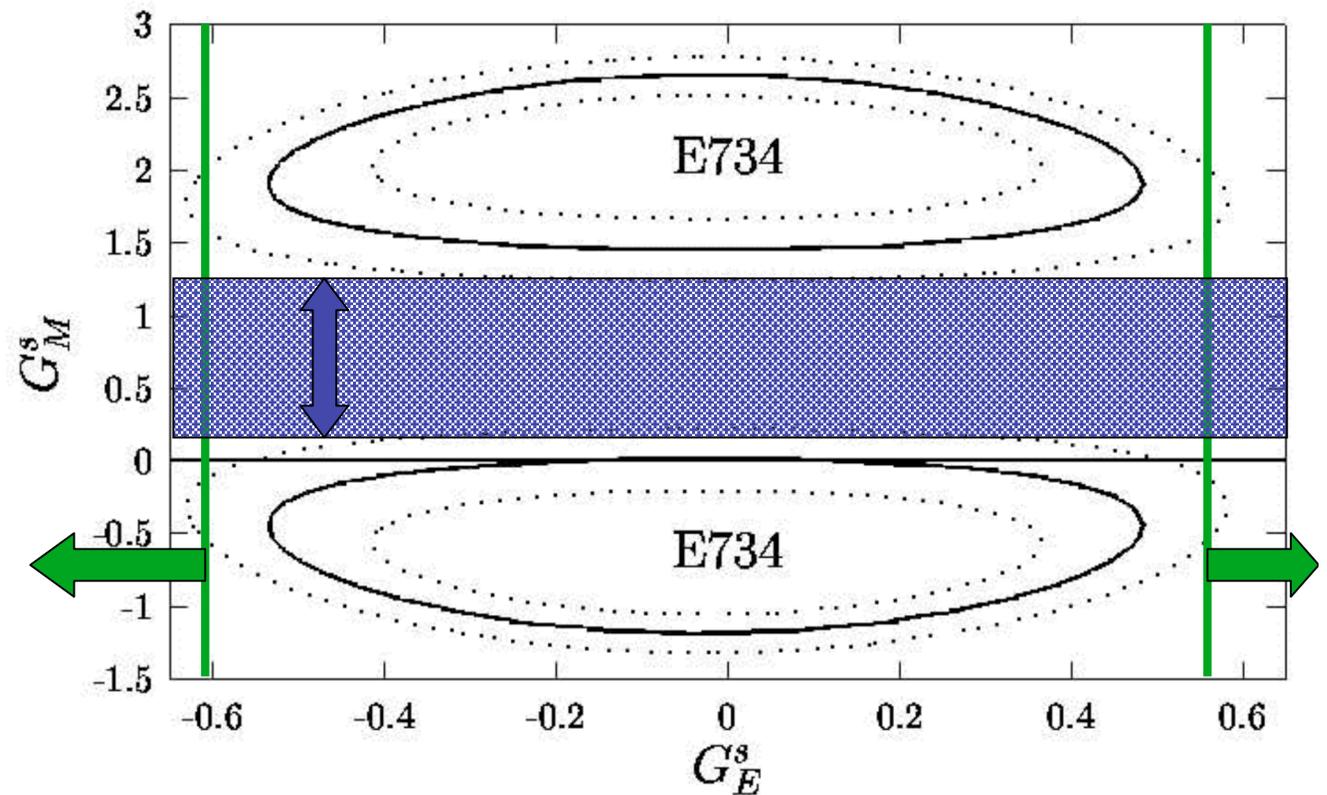
$$c = \frac{16\pi}{W} \frac{E_\nu^2}{Q^2} \frac{\Delta}{G_F^2} - ab$$



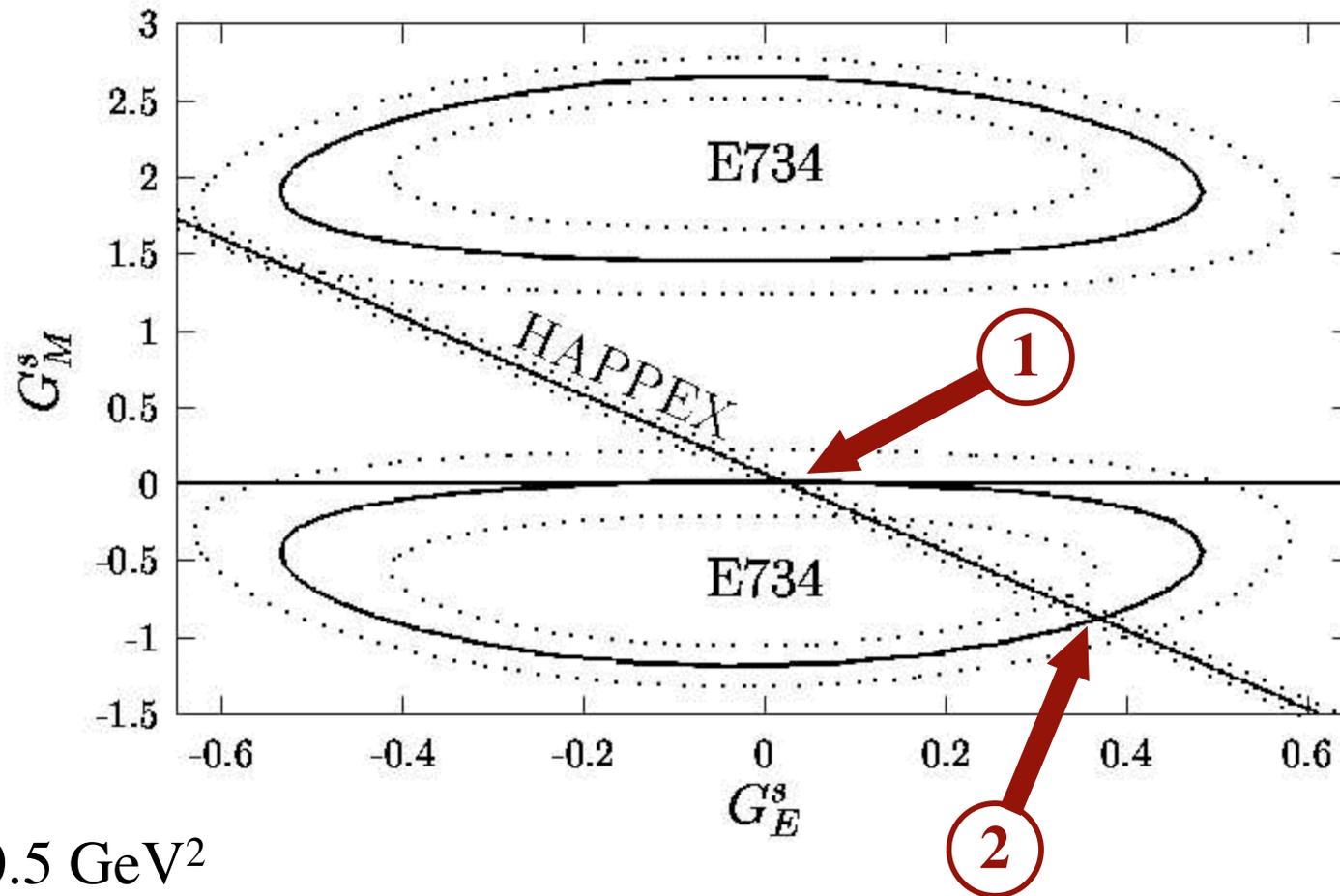
# What you learn from the $\Sigma$ expression

- multi-valued function of strange electric and magnetic form factors
- At this  $Q^2$ :
  - rules out  $|G_E^s| > \sim 0.6$
  - rules out moderate positive values of  $G_M^s$

$$Q^2 = 0.5 \text{ GeV}^2$$



# Combining E734 and HAPPEX

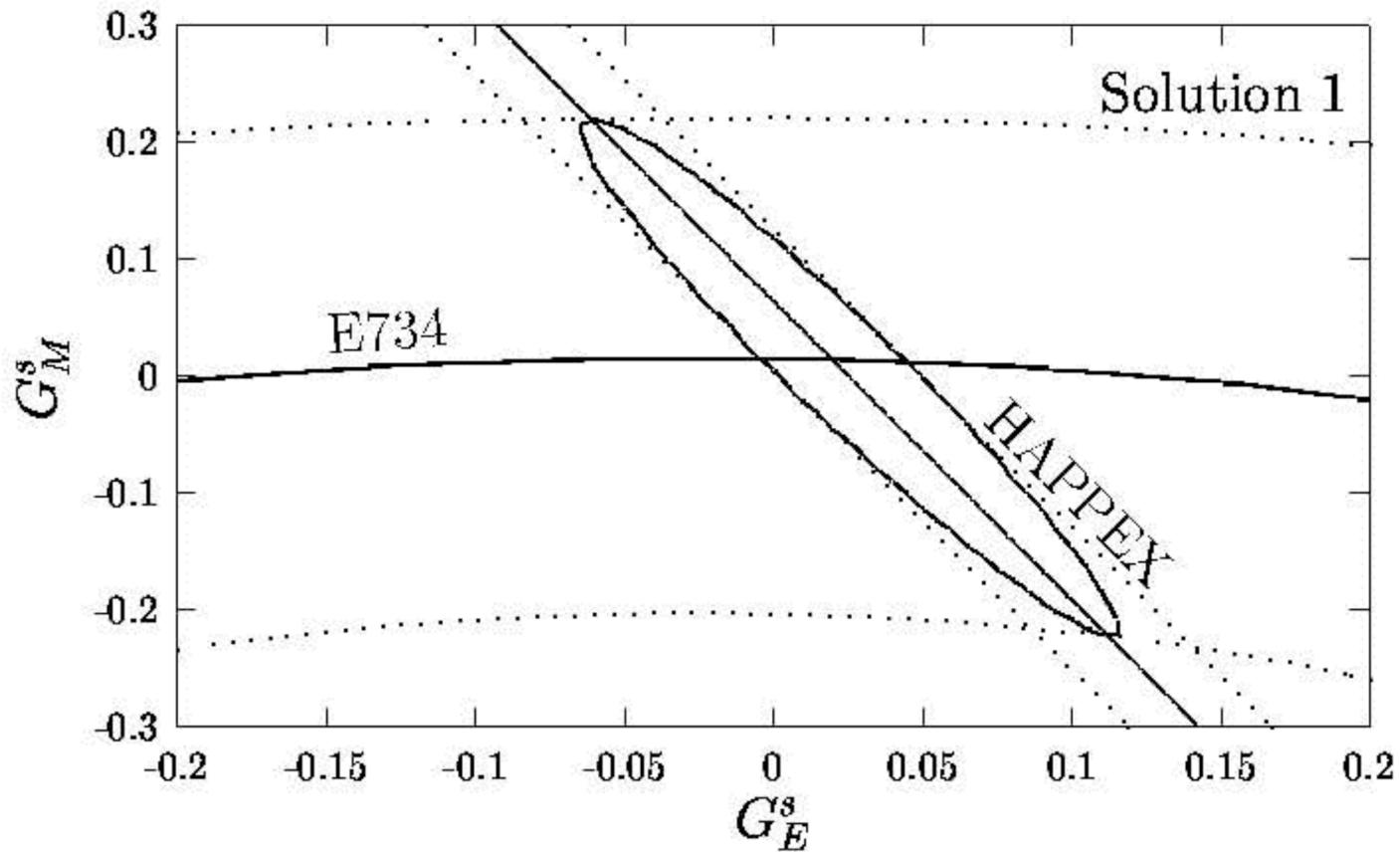


$$Q^2 = 0.5 \text{ GeV}^2$$

There are two solutions!

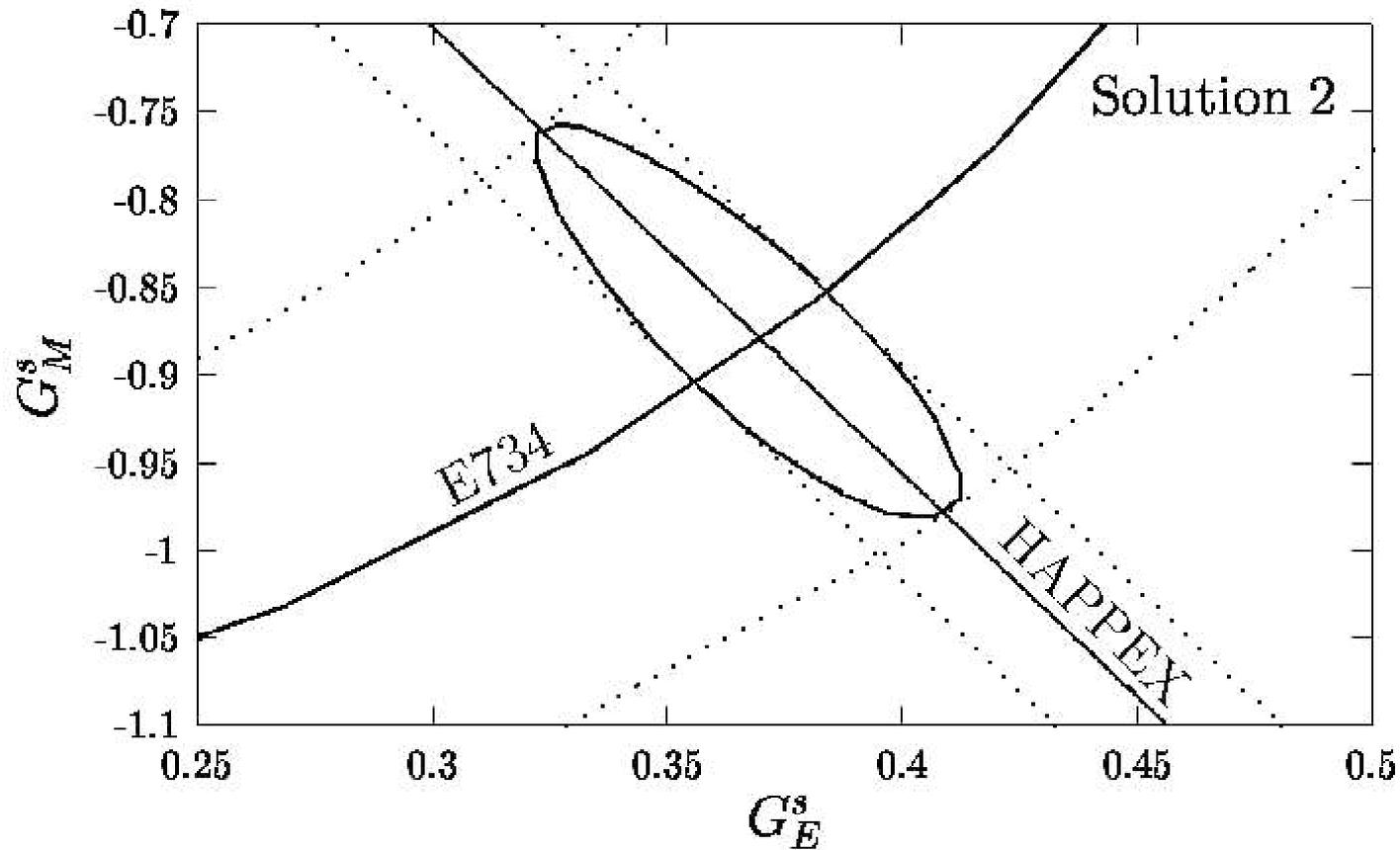
# Solution 1

$G_E^s$	$0.02 \pm 0.09$
$G_M^s$	$0.00 \pm 0.21$
$G_A^s$	$-0.09 \pm 0.05$



# Solution 2

$G_E^s$	$0.37 \pm 0.04$
$G_M^s$	$-0.87 \pm 0.11$
$G_A^s$	$0.28 \pm 0.10$



## The Two Solutions

$$Q^2 = 0.5 \text{ GeV}^2$$

	Solution 1	Solution 2
$G_E^s$	$0.02 \pm 0.09$	$0.37 \pm 0.04$
$G_M^s$	$0.00 \pm 0.21$	$-0.87 \pm 0.11$
$G_A^s$	$-0.09 \pm 0.05$	$0.28 \pm 0.10$

### Three reasons to prefer Solution 1:

- $G_A^s$  in Solution 1 is consistent with DIS estimate at  $Q^2 = 0$
- $G_M^s$  in Solution 1 is consistent with SAMPLE result at  $Q^2 = 0.091 \text{ GeV}^2$
- $G_M^s$  in Solution 1 is consistent with the Lattice QCD (Leinweber et al.) prediction of  $G_M^s(Q^2 = 0) = -0.051 \pm 0.021$ .

Additional measurement (JLab E91-004 or  $G^0$ -Backward) needed to determine the correct solution at this  $Q^2$ .

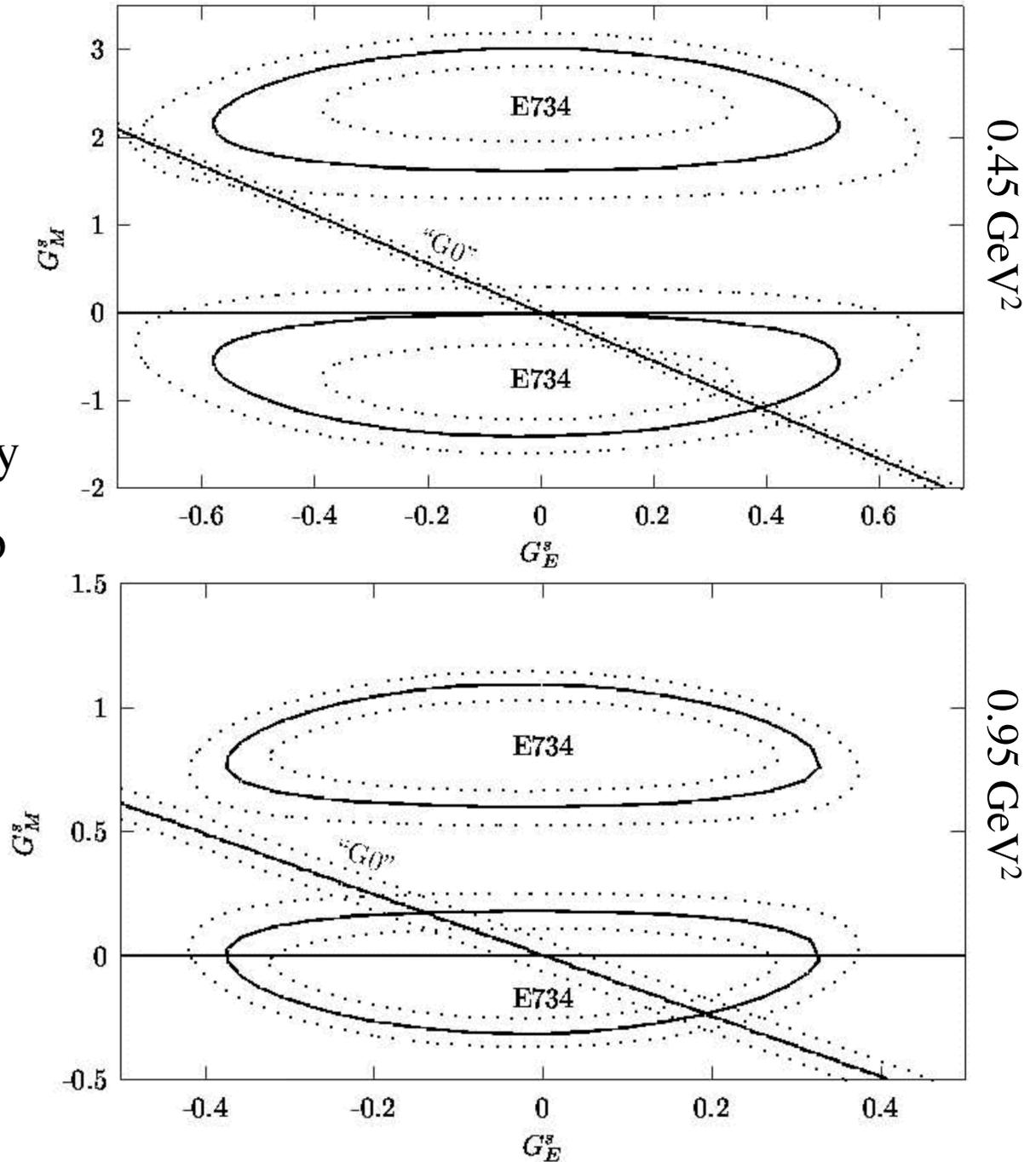
# Combining E734 and $G^0$ (Forward)

- First phase of  $G^0$  is similar to HAPPEX: forward electron scattering, thus little sensitivity to  $G_A^e$ .
- Get linear combinations of  $G_E^s$  and  $G_M^s$  at four  $Q^2$  points (0.45, 0.55, 0.75, 0.95 GeV<sup>2</sup>) in same range as E734.
- $G_A^e$ 
  - Use a value for  $G_A^e$  from Zhu et al., OR
  - Just set  $G_A^e = 0 \pm 1$  and live with slightly wider uncertainty band.
- Similar analysis as for HAPPEX will give two solutions at each  $Q^2$  point.
- First  $G^0$ (Backward) measurement (or JLab E91-004 measurement) will select correct solution set.

# What it might look like...

"Solutions" produced by forcing the  $G^0$  "data" to go through the origin (like HAPPEX), and using  $G_A^e = 0 \pm 1$ .

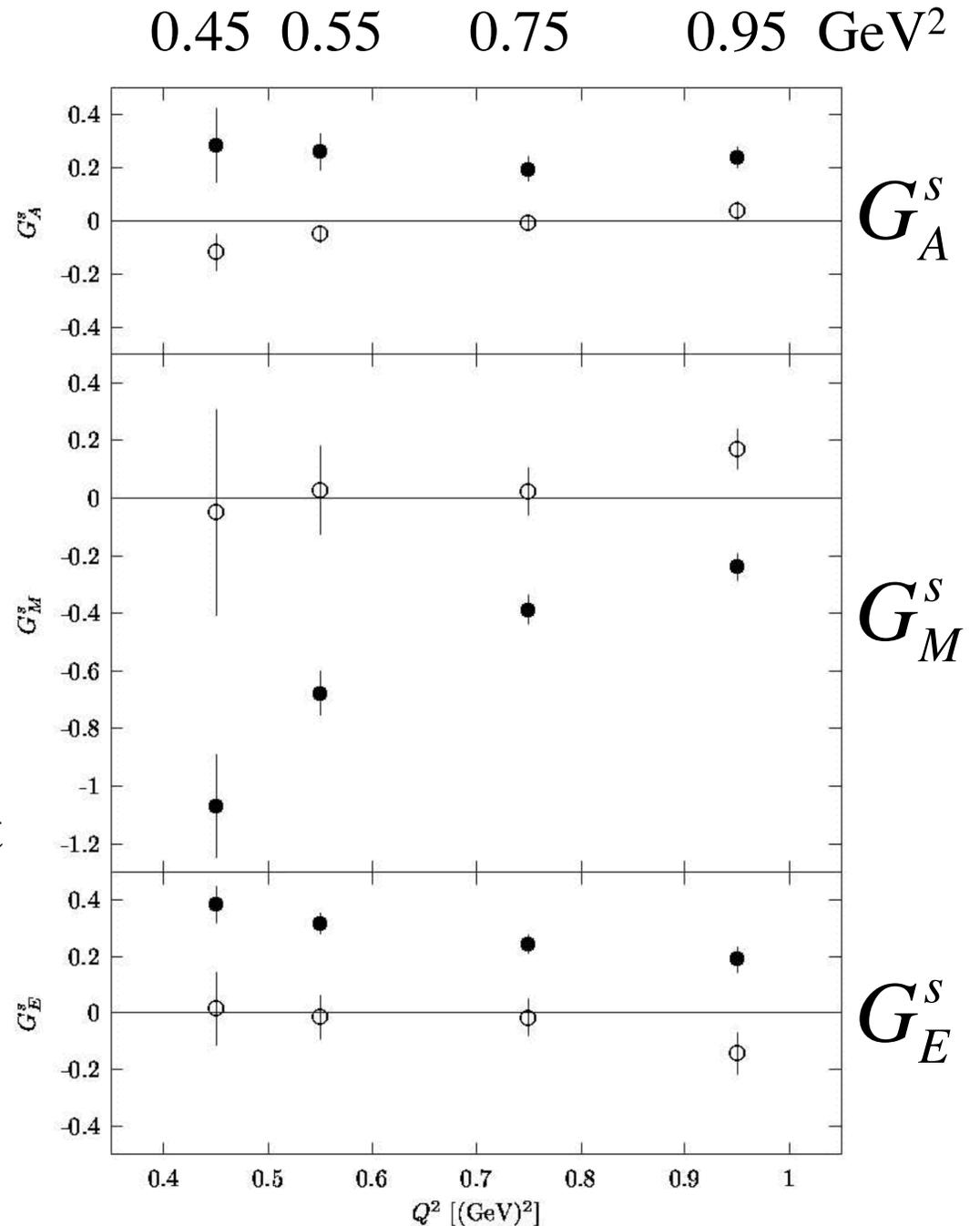
$$G_E^s + \alpha G_M^s = 0 \pm \beta$$



# E734 + $G^0$ (Forward) expected uncertainties

- Solution 1
- Solution 2

“Data” are clearly arbitrary, but  
sizes of uncertainties are not.



# A future experiment to determine $\Delta s$

Even if the program I have described determines the strange axial form factor down to  $Q^2 = 0.45 \text{ GeV}^2$  successfully, it almost certainly will not determine the  $Q^2$ -dependence sufficiently for an extrapolation down to  $Q^2 = 0$ .

Also, questions remain about the normalization of the E734 data. Most of their target protons were inside of carbon nuclei, and there was not much known about nuclear transparency in the mid-1980's. The E734 collaboration **did** make a correction for transparency effects, but this issue needs to be revisited if we continue to use the E734 data. (I have the original E734 simulation code and am working on this project with a student this summer.)

A new experiment has been proposed to measure elastic and quasi-elastic neutrino-nucleon scattering to sufficiently low  $Q^2$  to measure  $\Delta s$  directly.

# FINeSSE

A Proposal for a Near Detector Experiment on the Booster  
Neutrino Beamline:  
FINeSSE: Fermilab Intense Neutrino Scattering Scintillator  
Experiment

November 23, 2003

Twofold proposal:

- 1) Improve (with MiniBooNE) measurements of neutrino mixing phenomena
- 2) Measure the strange axial form factor down to  $Q^2 = 0.2 \text{ GeV}^2$

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# FINeSSE Determination of $\Delta s$

Measure ratio of NC to CC neutrino scattering from nucleons:

$$R_{\text{NC/CC}} = \frac{\sigma(\nu p \rightarrow \nu p)}{\sigma(\nu n \rightarrow \mu^- p)}$$

⇒ Numerator is sensitive to  $(-G_A^{CC} + \boxed{G_A^S})$

⇒ Denominator is sensitive to  $G_A^{CC}$  only

⇒ Both processes have unique charged particle  
final states signatures

⇒ Ratio largely eliminates uncertainties in neutrino flux,  
detector efficiency, and nuclear target effects

A 6% measurement of  $R_{\text{NC/CC}}$  down to  $Q^2 = 0.2 \text{ GeV}^2$   
provides a  $\pm 0.04$  measurement of  $\Delta s$ .

## In conclusion...

- Get values for all three strange form factors in the range  $0.45 < Q^2 < 0.95 \text{ GeV}^2$  by combining BNL E734 neutrino scattering data with JLab  $G^0$  electron scattering data

(this work available at PRL 92 (2004) 082002 and hep-ex/0310052)

- FINeSSE the job by measuring the NC/CC ratio in neutrino scattering to sufficiently low  $Q^2$  to get all three strange form factors over the  $Q^2$  range  $0.2 < Q^2 < 1.0 \text{ GeV}^2$ .

 determine  $\Delta s$



Non dimenticare di misurare  $\Delta s$  !